PHYS 1210 Discussion 2

Brief Solutions

1. Monty Python and the Siege of Castle Aarrggh

The specifics of the situation are an initial height $y_0 = h = 40.0$ m, an initial speed of 20.0 m/s, and an initial direction of 20°. We can then automatically set up the projectile kinematic equations.

Components of initial velocity: $v_{0x} = v_0 \cos\theta = 18.79$ m/s, $v_{0y} = v_0 \sin\theta = 6.84$ m/s¿

Horizontal (x direction):

$$a_x = 0$$
$$v_x = v_{0x}$$
$$x = v_{0x}t$$

Vertical (y direction):

$$a_y = -g$$
$$v_y = v_{0y} - gt$$
$$y = h + v_{0y}t - gt^2/2$$

a. Time to travel 60 meters horizontally

Here we're concerned with the horizontal position x, so we solve $x = v_{0x}t$ for t to give $t = x/v_{0x} = 3.19$ s.

b. Is Arthur hit?

To answer this question, we need to know how high above the ground the cow is when it reaches Arthur. If it is above about 2 meters, it goes over his head. If it is below about -1 meters, it lands in front of him. (I'm not saying below zero alone is good enough, because cows aren't points, and a cow landing right in front of Arthur would still slide into him.) To answer the question, we substitute the time form part a into the y equation: $y = h + v_{0y}t - gt^2/2 = 11.89$ m, well over his head.

c. Time for cow to land on the ground

Now we need to solve the y equation to find the time t at which $y = 0$. The y equation is quadratic in t, so we need to use the quadratic formula. This gives *two* solutions, one negative and one positive. We want the positive solution, because the kinematic equations don't apply before $t = 0$. The cow hits the ground at 3.64 s.

d. Cow's landing speed

At landing time, we find the cow's speed. That's the magnitude of its velocity, which we find using the Pythagorean Theorem with the components v_x and v_y. We don't need to find a formula for v_x, because it doesn't change. The formula for $v_{0y} =_{0y} -gt$ is above, giving -28.2 m/s. Thus $v = \sqrt{v_0x^2 + (v_{0y} - gt)^2} = 34.4$ m/s.

e. Cow's terminal angle

For this, we can use the arctangent, $\theta = \arctan(v_y/v_x) = -57°$. The negative value means that the angle is clockwise of the $+x$ axis, so the direction is 57 degrees below horizontal.

2. Centrifuge

In this scenario we characterize the uniform circular motion by the angular speed ω and radius R. At any time, the angular position is $\theta = \omega t$, the tangential speed is ωR, and the acceleration is $\omega^2 R$ toward the center.

a. Components of position

$x = R\cos(\omega t)$, $y = R\sin(\omega t)$.

b. Components of velocity

$v_x = dx/dt = -\omega R\sin(\omega t)$; $v_y = dy/dt = \omega R\cos(\omega t)$

c. Velocity in polar coordinates

This is tough using the formulas directly. The magnitude is straightforward; $v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(\omega R)^2(\cos^2(\omega t) + \sin^2(\omega t))} = \omega R$.

The direction gives $\arctan(v_y/v_x) = \arctan(-\cot(\omega t)) = \omega t + \pi/2$.

d. Speed

The magnitude of velocity is ωR.

e. Position and velocity directions

Velocity leads position by $90°$.

f. Components of acceleration

$a_x = dv_x/dt = -\omega^2 R\cos(\omega t)$; $a_y = dv_y/dt = -\omega^2 R\sin(\omega t)$.

g. Magnitude of acceleration

$a = \omega^2 R$.

h. Direction of acceleration

Acceleration is in the opposite direction of position; we can see this by the negative signs in the formulas for the components.

i. Specific example: Brooks Centrifuge

$R = 9.50$ m, $\omega = 3.04$ rad/s.

I. Speed

$v = \omega R = 28.88$ m/s.

II. Acceleration

$a = \omega^2 R = 87.80$ m/s^2.

III. g-value of acceleration

$87.80\ \text{m/s}^2 \cdot \frac{g}{9.8\ \text{m/s}^2} = 8.96$ g