PHYS 1210 Discussion 4

Brief Solutions

1. Kaden on an elevator

This is almost too simple for a discussion section. But it's good to reinforce fundamentals.

The first step is to make a free body diagram. Unfortunately, graphics are difficult for me to display here, but the fbd is pretty simple: weight downward, and a normal force upward for every single case in this scenario. The only difference is in the normal force, as Kaden's weight is just $mg = 588$ kg every time.

All the questions ask about the reading on the scale. The scale registers the magnitude of the normal force exerted on Kaden, or the normal force he exerts on the scale.

a. Elevator at rest

Acceleration is zero, so the normal force $= mg = 588$ N.

b. Accelerating upward at 1.4 m/s^2

Net force $m\vec{a} = m\vec{g} + \vec{N}$, so $\vec{N} = m(\vec{a} - \vec{g})$. Paying attention to directions, with positive being upward gives us $N = m(a + g) = (60 \text{ kg})(1.4 + 9.8) \text{ m/s}^2 = (60)(11.2) \text{ kg} \cdot \text{m/s}^2 = 672$ N.

c. Constant ascent

Velocity is constant, so acceleration is zero, and $N = 588$ N.

d. Ascending with slowing

Acceleration is now *downward*, and $\vec{N} = m(\vec{a} - \vec{g})$ gives us $N = m(-a + g) = (60 \text{ kg})(-1.4 + 9.8) \text{ m/s}^2 = (60 \cdot 8.4) \text{ N} = 504$ N.

e. At rest at top floor

The scale reads Kaden's weight of 588 newtons.

f. Descending, speeding up

Acceleration is downward; normal force is 504 newtons.

g. Descending, constant velocity

Net force is zero, so the scale reads Kaden's weight of 588 newtons.

h. Descending, slowing

Acceleration is upward, so normal force is grerater in magnitude thatn Kaden's weight. The scale reads 672 newtons.

2. Amusement park rotor

a. Free body diagram

Three possible forces here: weight straight downward, normal force perpendicular to the wall (in this diagram, mostly to the left and a little bit upward), and possibly friction. Depending on how large the normal force is, friction may need to be downward!

The *net* force should be purely horizontal, because the rider is accelerating toward the center of his circular path. The wall is banked a little to not ned to rely entirely on friction to prevent sliding downward.

b. Net force direction

As explained in justifying the fbd, the net force is horizontal, to the left.

c. Function of each quantity

Coefficient of friction μ: The larger μ is, the easier it will be to keep the rider in the ride.

Angular speed ω: The faster the rotor spins, the greater the normal force. This increases the friction force and increases the upward component of the normal force, both of which lift the rider against gravity.

Wall tilt α: The larger α is, the greater the upward component of the normal force will be, lifting the rider. To the logical extreme, if $\alpha = 90°$, the rotor won't need to spin at all or have nay friction to keep the rider inside!

Rider's mass: This has no effect other than requiring more energy to get the machine up to speed, and also raising concerns about balancing the rotor.

Radius R: For the same rotation al speed, a larger radius gives a greater centripetal acceleration, and consequently a greater normal force. So the larger the radius, the easier it will be to keep riders inside.

3. Sliding blocks

For both blocks, the normal force is a force of constraint, keeping the blocks on the track. Block 1 moves and accelerates only in the horizontal direction, while block 2 moves and accelerates only inclined at an angle θ off horizontal.

a. Acceleration

For block 1, $m_1 a = T - \mu_1 m_1 g$. For block 2, $m_2 a = m_2 g \sin\theta - \mu_2 m_2 g \cos\theta - T$. These are two equations in two unknowns, a and T. We are looking for a, not T, so

$$\begin{aligned} T &= m_1 a + \mu_1 m_1 g \\ m_2 a &= m_2 g \sin\theta - \mu_2 m_2 g \cos\theta - m_1 a - \mu_1 m_1 g \\ m_2 a + m_1 a &= m_2 g \sin\theta - \mu_2 m_2 g \cos\theta - \mu_1 m_1 g \\ a(m_1 + m_2) &= g(m_2 \sin\theta - \mu_2 m_2 \cos\theta - \mu_1 m_1) \\ a &= g\frac{m_2 \sin\theta - \mu_2 m_2 \cos\theta - \mu_1 m_1}{m_1 + m_2} \end{aligned}$$

b. With numbers

Substituting the given numbers in, I get $a = 0.317$ m/s^2.

c. Angle giving zero acceleration

The function to be zeroed (find the root) can be simplified a little.

$$\begin{aligned} 0 = a &= g\frac{m_2 \sin\theta - \mu_2 m_2 \cos\theta - \mu_1 m_1}{m_1 + m_2} \\ 0 &= m_2 \sin\theta - \mu_2 m_2 \cos\theta - \mu_1 m_1 \end{aligned}$$

Trigonometry does not come to my rescue here, as I can't transform this to a single trig function I can invert. I stopped refining the solution at $\theta = 46.05°$.