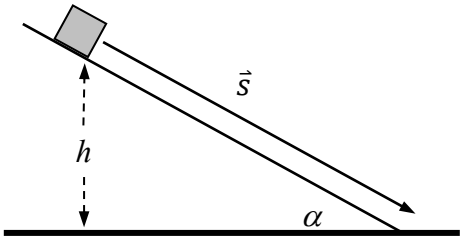
PHYS 1210 Worksheet 5. Work Solutions

A block of mass m is released from rest on a frictionless ramp at an angle α and height h. The block slides down the ramp, along a displacement vector $\vec{s}$ ending at the bottom of the ramp. The forces acting on the block are its weight $\vec{w}$ and the normal force $\vec{N}$.



1. Next to the diagram of the block on the ramp, draw a free-body diagram for the block.
2. Find a formula for s, the magnitude of $\vec{s}$, in terms of h and α.

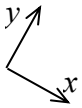
   $h/s = \sin(\alpha)$, so $s = h/\sin(\alpha)$.

3. Find a formula for x, the horizontal component of $\vec{s}$, also in terms of h and α.

   $h/x = \tan(\alpha)$, so $x = h/\tan(\alpha)$.

4. Analyze the system using both inclined (parallel and perpendicular to the ramp) and plumb (vertical and horizontal) coordinates. For each set of coordinates, find the x- and y-components of the indicated vectors.

Inclined coordinates



$w_x = +mg\sin(\alpha)$

$w_y = -mg\cos(\alpha)$

$N_x = 0$

$N_y = +mg\cos(\alpha)$

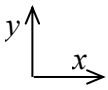
$\Sigma F_x = +mg\sin(\alpha)$

$\Sigma F_y = 0$

$s_x = s = h/\sin(\alpha)$

$s_y = 0$

Plumb coordinates



$w_x = 0$

$w_y = -mg$

$N_x = +N\sin(\alpha) = +mg\cos(\alpha)\sin(\alpha)$

$N_y = +N\cos(\alpha) = +mg\cos^2\alpha$

$\Sigma F_x = +mg\cos(\alpha)\sin(\alpha)$

$\Sigma F_y = mg\cos^2\alpha - mg = -mg\sin^2\alpha$

$s_x = x = h/\tan(\alpha)$

$s_y = -h$

4. Find the indicated dot products from the vector components.

Inclined coordinates

$\vec{w} \cdot \vec{s} = mg \sin(\alpha) \cdot h/\sin(\alpha) + 0 = +mgh$

$\vec{N} \cdot \vec{s} = 0$

$\Sigma\vec{F} \cdot \vec{s} = mg \sin(\alpha) \cdot h/\sin(\alpha) = +mgh$

Plumb coordinates

$\vec{w} \cdot \vec{s} = +mgh$

$\vec{N} \cdot \vec{s} = mgh \cos^2(\alpha) - mgh \cos^2(\alpha) = 0$

$\Sigma\vec{F} \cdot \vec{s} = mgh \cos^2(\alpha) + mgh \sin^2(\alpha) = mgh$