PHYS 1210 Discussion 7. Momentum

Brief Solutions

1. Puck-Stick Collision

We are given the masses of the puck ($m_p = 0.175$ kg) and stick (0.455 kg), and their initial velocities $\vec{v}_{pi} = 25.0\ \hat{\imath}$ m/s and $v_{si} = 0$.

A. Total initial momentum

The stick initially is at rest, so the total is just the momentum of the puck, $m_p\vec{v}_{pi} = (4.375\ \hat{\imath})$ kg $\cdot$ m/s.

B. Conservation of momentum

The stick and puck receive no outside forces during the collision, so momentum is conserved.

C. Total final momentum

Because momentum is conserved, total final momentum is the same as total initial momentum.

D. Final momentum of the puck

We know the final velocity $\vec{v}_{pf} = (5.5\ \hat{\imath} + 9.526\ \hat{\jmath})$ m/s of the puck, so its momentum then is just its mass times its velocity, $\vec{p}_{pf} = m_p\vec{v}_{pf} = (0.963\ \hat{\imath} + 1.667\ \hat{\jmath})$ kg $\cdot$ m/s.

E. Final momentum of the stick

The final total momentum is the sum of the final momentum of the puck and stick. We know the final total momentum, and we know the final momentum of the puck, so the final momentum of the stick is the difference.

$$\begin{aligned}\vec{p}_f &= \vec{p}_{pf} + \vec{p}_{sf}\\ \vec{p}_{sf} &= \vec{p}_f - \vec{p}_{pf}\\ &= 4.375\ \hat{\imath}\ \text{kg}\cdot\text{m/s} - (0.9625\ \hat{\imath} + 1.667\ \hat{\jmath})\ \text{kg}\cdot\text{m/s}\\ &= (3.4235\ \hat{\imath} - 1.667\ \hat{\jmath})\ \text{kg}\cdot\text{m/s}\end{aligned}$$

F. Final velocity of the stick

Divide its final momentum by it mass. $\vec{v}_{sf} = (7.5\ \hat{\imath} - 3.67\ \hat{\jmath})$ m/s.

G. Conservation of kinetic energy

The initial kinetic energy of the system is 54.7 joules; the final kinetic energy is 26.4 joules. Kinetic energy is not conserved.

H. Conservation of total mechanical energy

We have no indication that the lost kinetic energy became potential energy, so mechanical energy is not conserved.

I. Type of collision

This is an inelastic collision.

2. Impulse of the collision

A. Impulse on the puck

The impulse-momentum theorem tells us that the momentum change of the puck is equal to the impulse that acted on the puck. The puck's momentum change is $(-3.413\hat{\imath} + 1.667\hat{\jmath})$ kg $\cdot$ m/s.

B. Average force on the puck

Impulse is force applied along a distance, $J = \int F dt$; assuning a constant force, this becomes $F\Delta t$. Thus $F = J/\Delta t = (-75.83\hat{\imath} + 37.05\hat{\jmath})$ N. The magnitude of this force is 84.4 N. The direction, if anyone cares, is 154°. Here, you must take care not to blindly trust the result of the inverse tangent function, because the vector is in quadrant 2.

3. Center of mass

We are given two masses: $m_A = 2.063\ m_S$ and $m_B = 1.018\ m_S$, and a separation of $D = 2.049 \times 10^{12}$ m.

The question asks for center of mass distance from star A, so it makes sense to set the origin there. Thus $x_A = 0$, $x_B = D$. Then

$$\begin{aligned} x_C &= \frac{x_A m_A + x_B m_B}{m_A + m_B} \\ &= \frac{0 + D m_B}{m_A + m_B} \\ &= (2.049 \times 10^{12}\ \text{m}) \frac{1.018}{2.063 + 1.018} \\ &= 7.96 \times 10^{11}\ \text{m} \end{aligned}$$

Collision Problem Worksheet

Mass of puck m_p = 0.175 kg Mass of stick m_s = 0.455 kg

What	*x*	*y*
Initial velocity of puck v_{pi}	25.0 m/s	0 m/s
Initial velocity of stick v_{si}	0 m/s	0 m/s
Initial momentum of puck p_{pi}	4.375 kg·m/s	0
Initial momentum of stick p_{si}	0	0
Initial total momentum Σp_i	4.375 kg·m/s	0
Final total momentum Σp_f	4.375 kg·m/s	0
Final velocity of puck v_{pf}	11.0 m/s cos(60°) = 5.5 m/s	11.0 m/s sin(60°) = 9.53 m/s
Final momentum of puck p_{pf}	0.9625 kg·m/s	1.667 kg·m/s
Final momentum of stick p_{sf}	3.4125 kg·m/s	-1.667 kg·m/s
Final velocity of stick v_{sf}	7.5 m/s	-3.66 m/s
Initial kinetic energy of puck K_{pi}	54.69 J	
Initial kinetic energy of stick K_{si}	0	
Initial total kinetic energy ΣK_i	54.69 J	
Final kinetic energy of puck Kp_f	10.59 J	
Final kinetic energy of stick Ks_f	15.85 J	
Final total kinetic energy ΣK_f	26.44 J	