PHYS 1210 Discussion 11. Waves

Brief Solutions

1. Wave with amplitude A, frequency f, and speed v

A. Period $T = 1/f$.

B. Wavelength $\lambda = v/f$.

C. Angular wavenumber $k = 2\pi/\lambda = 2\pi f/v$.

D. Angular frequency $\omega = 2\pi/T = 2\pi f$.

E. The equation is $y = A\cos\left(2\pi(f/v)x - 2\pi f + \varphi\right)$. The phase offset φ is undetermined, but necessary to include, because we don't know the initial phase.

2. Wave given amplitude, wavelength, and velocity,

A. The frequency is $f = v/\lambda = (3.20\ \text{m/s})/(1.6\text{m}) = 2.00$ Hz.

B. The period is $T = 1/f = 0.500$ s.

C. The angular frequency is $\omega = 2\pi f = 4\pi$ rad/s ≈ 12.6 rad/s.

D. The angular wavenumber is $k = 2\pi/\lambda = 1.25\pi$ rad/m ≈ 3.93 rad/m.

E. The equation is $y = (20\ \text{cm})\cos\left((1.25\pi\ \text{rad/m})x - (4\pi\ \text{rad/s})t\right)$.

3. Standing wave

A. Add the two functions

$$\begin{aligned}
y &= A\cos(kx - \omega t) + A\cos(kx + \omega t) \\
&= A\left(\cos(kx - \omega t) + \cos(kx + \omega t)\right) \\
&= A\left(\cos(kx)\cos(-\omega t) - \sin(kx)\sin(-\omega t) + \cos(kx)\cos(\omega t) - \sin(kx)\sin(\omega t)\right) \\
&= A\left(\cos(kx)\cos(\omega t) + \sin(kx)\sin(\omega t) + \cos(kx)\cos(\omega t) - \sin(kx)\sin(\omega t)\right) \\
&= 2A\cos(kx)\cos(\omega t)
\end{aligned}$$

This function is two sinusoidal functions multiplied together. The first is a function of space (position) and the second is a function of time. This means that the function of space stays in place, but its vertical extent varies in time. The time function is sinusoidal, so the standing function oscillates.

B. Repeat distance

The repeat distance λ is the change in x giving the phase kx incremented by 2π, $k(x+\lambda) = kx + 2\pi$, so $\lambda = 2\pi/k$.

C. Repeat time

The repeat time T is the change in t giving the phase ωt incremented by 2π, $\omega(t+T) = \omega t + 2\pi$, so $T = 2\pi/\omega$.

4. Beats

A. Sum of the two functions

$$
\begin{aligned}
y &= A\cos\left([\omega+\delta]t\right) + A\cos\left([\omega-\delta]t\right) \\
&= A\cos(\omega t+\delta t) + A\cos(\omega t-\delta t) \\
&= A\cos(\omega t)\cos(\delta t) - A\sin(\omega t)\sin(\delta t) + A\cos(\omega t)\cos(-\delta t) - A\sin(\omega t)\sin(-\delta t) \\
&= A\cos(\omega t)\cos(\delta t) - A\sin(\omega t)\sin(\delta t) + A\cos(\omega t)\cos(\delta t) + A\sin(\omega t)\sin(\delta t) \\
&= 2A\cos(\omega t)\cos(\delta t)
\end{aligned}
$$

This is two sinusoidal functions multiplied together. Both are functions of time, but with different frequencies. One has a frequency of $f_1 = \omega/2\pi$, and the other has a frequency of $f_2 = \delta/2\pi$.

B. Frequency

This has not one, but two frequencies. If they are very different, say f_1 is a much higher frequency than f_2, then it will be a tone with a frequency of f_1 oscillating in loudness with frequency f_2. Frequency $f_!$ is the average frequency of the two component tones, and frequency f_2 is half their difference.

C. Variation of displacement with time

Essentially, this is a tone with frequency $f_1 = \omega/2\pi$ will have an amplitude varying with frequency $f_2 = \delta/2\pi$.