PHYS 1210 Discussion 12

Brief Solutions

1. Grand piano wire

We are given that frequency $f = 27.5$ Hz, length $L = 2.000$ m, tension $F = 1000$ N. This looks like we are being set up to use the formula for propagation speed $v = \sqrt{F/\mu}$.

A. Length density μ

Sure enough, we need to use that formula, but we haven't been told v, which we need to find μ. We can find it from the wave speed formula $v = \lambda f$; we know f already, so we need to find the wavelength λ. We can get that from knowledge of the normal modes of a string. The fundamental mode of the string wave has a frequency of 27.5 Hz, and a wavelength of twice the string length: $\lambda = 4.00$ m.

$$v = \sqrt{F/\mu}$$
$$v^2 = F/\mu$$
$$\mu = F/v^2 = \frac{F}{(\lambda f)^2} = \frac{1000\ \text{N}}{\big[(4.000\ \text{m})(27.5\ \text{Hz})\big]^2} = 0.08264\ \text{kg/m}$$

B. Diameter of the string

The diameter and the density must have something to do with the length density. The simplest way to pick this out is to see that the density and mass of the string will tell us its volume, and the volume and length will tell us its diameter.

First, the volume. Density is mass per volume, $\rho = m/V$. Length density is mass per length, $\mu = m/L$, so mass is $m = \mu L$, and we know both μ and L. A piano wire is basically a long cylinder whose volume is its length times its cross sectional area:

$$\begin{aligned}
V &= \pi r^2 L \\
V &= m/\rho \\
\pi r^2 L &= m/\rho \\
\pi r^2 L &= \mu L/\rho \\
\pi r^2 &= \mu/\rho \\
r^2 &= \frac{\mu}{\pi\rho} \\
&= \frac{(0.08264\ \text{kg/m})}{\pi(7800\ \text{kg/m}^3)} \\
&= 3.3726 \times 10^{-6}\ \text{m}^2 \\
r &= 1.84 \times 10^{-3}\ \text{m} = 1.84\ \text{mm}
\end{aligned}$$

The diameter is twice the radius, or 3.67 mm. That's a pretty thick wire! In fact, it's more a steel *rod* than it is a wire.

C. Flexibility

It won't have the transverse flexibility we assumed when we derived the formula for transverse wave propagation speed in the first place. This would give us different normal modes and thus different acoustic behavior.

D. Piano wire construction

Heavy piano wires are made with a heavy wire helically wound around a thinner central tension wire. This gives the necessary length density without sacrificing flexibility.

2. Formula 1 race car speed and pitch, coming and going

When the car approaches, the detected frequency is 380 Hz; when the car recedes, the detected frequency is 240 Hz. We are told that the speed of sound is 342 m/s. We assume that the approach and recessional speeds are the same.

In both cases, the Doppler shift formula applies:

$$f_D = f_S \frac{v - v_D}{v - v_S}$$

In both cases, the detector velocity v_D is zero. The source velocity v_S has the same magnitude in both cases, but opposite signs. We have

$$f_1 = f_S \frac{v}{v - v_S} \qquad f_2 = f_S \frac{v}{v + v_S}$$

These are two equations in two unknowns, f_S and v_S.

A. Car's speed v_S

The simplest way to find this is to solve one of the equations for f_S and substitute it into the other equation, giving a single equation in the desired unknown v_S.

$$f_S = f_1 \frac{v - v_S}{v} \qquad f_S = f_2 \frac{v + v_S}{v}$$

$$f_1 \frac{v - v_S}{v} = f_2 \frac{v + v_S}{v}$$

$$f_1 v - f_1 v_S = f_2 v + f_2 v_S$$

$$v(f_1 - f_2) = v_S(f_1 + f_2)$$

$$v_S = v \frac{f_1 - f_2}{f_1 + f_2} = (342 \text{ m/s}) \frac{380 - 240}{380 + 240} = 77.22 \text{ m/s}$$

B. Car's frequency

Substituting

$$v_S = v \frac{f_1 - f_2}{f_1 + f_2}$$

into either

$$f_S = f_1 \frac{v - v_S}{v}$$

or

$$f_S = f_2 \frac{v + v_S}{v}$$

gives

$$f_S = 2 \frac{f_1 f_2}{f_1 + f_2} = 294 \text{ Hz}$$