PHYS 1210 Discussion 12

Brief Solutions

1. Hydraulic system

The specifics are that the internal pressure is 10 bar = 1,013,250 Pa, input force is 10 newtons, output load is 3.0 metric tons. The weight of 3 metric tons is 29,400 newtons.

A. Minimum diameter of output piston

The determining factor here is that pressure is force per area, $p = F/A$. The area determines the diameter by $A = \pi(D/2)^2$, so

$$p = F/A$$
$$A = F/p = 0.02902 \text{ m}^2$$

Then

$$A = \pi r^2$$
$$r = \sqrt{A/\pi}$$
$$d = 2r = 2\sqrt{A/\pi} = 19.22 \text{ cm}$$

B. Maximum diameter of input piston

Here it's the same pressure, with a different force. $A = F/p = (100 \text{ N})/(1,013,250 \text{ Pa}) = 0.987 \text{ cm}^2$. Then $d = 2\sqrt{A/\pi} = 0.560$ cm.

C. Responses to changes

If a heavier load is needed, the output (lift) cylinder will need to be wider. The input (drive) piston won't change, but each stroke will lift the load by a shorter distance.

If the hydraulic fluid can have a higher pressure, then the cross sections of the cylinders would be less (the cylinders would be narrower).

If a higher input force is possible, then the input (drive) piston could be wider. Each stroke could lift the load higher than before.

2. Test tube diver

The Cartesian diver floats because the buoyancy force exerted by the water on the diver (test tube and air bubble) is greater than its weight. When the bottle is squeezed, the pressure increases, collapsing the bubble. The collapsed bubble displaces less water, so the buoyancy force decreases.

3. Earth's atmosphere

We are told that sea level pressure is 101,325 pascals, and that Earth's radius is 6378 kilometers.

A. Weight of the atmosphere

Pressure is force per area, $p = F/A$. Here, the force is the weight of the atmosphere, and the area is the surface area of Earth. Solving for weight gives $F = pA$. The surface area of a sphere is $A = 4\pi r^2 = 5.112 \times 10^{14} \text{ m}^2$, giving $F = 5.180 \times 10^{19}$ newtons.

B. Mass of the atmosphere

Weight is $F = mg$, so mass is $m = F/g = 5.285 \times 10^{18}$ kg.

C. Correction for gravity decreasing with altitude

The weight of the air high in the atmosphere is less than the weight of air closer to the ground, because the gravitational field is higher at the ground. That means that the formula divides some of the weight by too high a field, giving a too-low estimate of mass. So the mass of the atmosphere must be a little more than we estimated.

D. Height of the atmosphere at constant density

If the atmosphere is thin, we can just estimate its volume as area·height, and not worry about subtracting spheres from each other. (You are free to do that if you like.) Density is mass per volume, $\rho = m/V$, so the volume of the atmosphere is $V = m/\rho = Ah$, so height $h = m/\rho A = 10100$ m $= 10.1$ km. Compared to the radius of Earth, that's thin.

4. Spaghettify

A. Gradient of the gravitational field

$$g = \frac{GM}{r^2}$$
$$dg/dr = \frac{-2GM}{r^3}$$

The negative sign just means that the field gets weaker farther away from the attractor.

B. Distance for a particular gradient

This is a seemingly strong gradient, a difference of a thousand Earth gravities between points just a meter apart. We just solve the formula above for r.

$$r^3 = \frac{-2GM}{dg/dr}$$
$$r = \sqrt[3]{\frac{-2GM}{dg/dr}} = 4.3 \text{ km}$$

C. Lateral field strength g_w/w

Ratios of sides of similar triangles gives us

$$\frac{g_w}{g} = \frac{w/2}{r} = \frac{w}{2r}$$
$$\frac{g_w}{w} = \frac{g}{2r}$$

Now, $g = GM/r^2$, so

$$\frac{g_w}{w} = \frac{GM/r^2}{2r} = \frac{GM}{2r^3}$$

This is very similar to the formula for the gradient. It's less by a factor of exactly 4. Correspondingly, $g_w/w = 2450$ (m/s^2)/m, which is 250 g's per meter. That is getting close to sufficient to elongate a person.