PHYS 1210 Discussion Worksheet 2
Motion in two dimensions

Concepts

Vectors of motion

Position $\vec{r} = (x, y, z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Velocity $\vec{v} = \frac{d\vec{r}}{dt} = (dx/dt\,, dy/dt\,, dz/dt) = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \left(dv_x/dt\,, dv_y/dt\,, dv_z/dt\right) = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

Component of $\vec{a}$ parallel to $\vec{v}$: rate of change of speed dv/dt

Component of $\vec{a}$ perpendicular to $\vec{v}$: affects direction only

Ballistic projectiles

When the only force is gravity (no air resistance, etc.), the horizontal (x) and vertical (y) components of the motion can be considered independently. For a projectile launched from (x_0, y_0) with initial speed v_0 at angle θ above horizontal, the initial velocity $\vec{v}_0 = v_0 \cos\theta\,\hat{\imath} + v_0 \sin\theta\,\hat{\jmath}$, and ½

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$
$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - ½gt^2$$

Problems

With your group, discuss how to answer the questions.

There is not space on this worksheet to work the rest of the questions. You will need your own scratch paper.

1. King Arthur stands before the Castle Aarrggh to demand the inhabitants give him the Holy Grail. Instead, the castle's defenders respond by throwing livestock at Arthur and his knights. In particular, they catapult a cow from a height of 40.0 meters above the ground at a speed of 20.0 meters per second and an initial angle of 20 degrees above horizontal. Arthur stands directly in the path of the hurtling cow, at a distance of 60.0 meters from the castle wall.
   a. How much time does the cow take to travel 60.0 meters horizontally from its launch?
   b. Does the cow hit Arthur? If not, does it land in front of him, or does it go over his head?
   c. How much time does the cow spend in the air before hitting the ground?
   d. With what speed does the cow hit the ground?
   e. Right before the cow hits the ground, what is the direction of its velocity? Give your answer as an angle below horizontal.

2. Space agencies and the military use large centrifuges to subject pilots in training to high accelerations. One such centrifuge has a radius of R. Simulating a climb, it rotates at an angular speed of ω. That means that at time t, the angular position θ of the trainee is $\theta = \omega t$.

   a. The trainee's position vector can be expressed as magnitude R and angle θ. Convert this to Cartesian components x and y.

   b. Differentiate x and y with respect to time to find the Cartesian components of the trainee's velocity $v_x = dx/dt$ and $v_y = dy/dt$.

   c. Convert the Cartesian components of velocity $\vec{v}$ to polar coordinates (magnitude and direction).

   d. What is the trainee's speed?

   e. How does the direction of the trainee's velocity $\vec{v}$ compare to their angular position $\theta = \omega t$?

   f. Differentiate v_x and v_y with respect to time to find the Cartesian components of the trainee's acceleration $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

   g. What is the magnitude of the trainee's acceleration?

   h. How does the direction of the trainee's acceleration compare to their angular position $\theta = \omega t$?

   i. The Brooks Centrifuge in San Antonio, TX has a radius R of 9.50 meters and its highest rotational speed for standard training is 3.04 radians per second.

      I. What is the trainee's speed in meters per second?

      II. What is the trainee's acceleration in meters per second per second?

      III. What is the trainee's acceleration as a multiple of the gravitational acceleration g?