PHYS 1210 Discussion 8. Moments of Inertia

1. What is the altitude of an equilateral triangle with sides of length a?

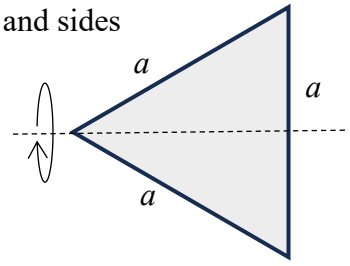
2. Frequently, we will be interested in the center of mass of an object when we contemplate its moment of inertia. In this worksheet, we will consider an equilateral triangle, so we will need to know the location of its center of mass. Explicitly finding that requires some double integration, and I don't want to get bogged down with that. Instead, we'll solve a similar problem with the same symmetry. Consider three equal point masses, one at each vertex of an equilateral triangle with sides of length a.
   A. How far is the center of mass from a side of the triangle? From a vertex of the triangle?
   B. What are the coordinates of the center of mass? (This depends, of course, on where you place the vertices of the triangle.)

3. What is the area of an equilateral triangle with sides of length a?

4. What is the area density σ (mass per unit area) of an equilateral triangle with mass M and sides of length a?

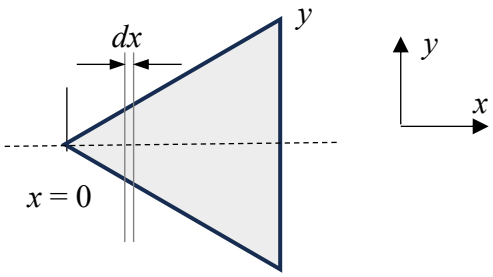
With all the preliminaries out of the way, we are finally ready to find some moments of inertia.

5. What is the moment of inertia of an equilateral triangle of mass M and sides of length a when rotated about a midline?



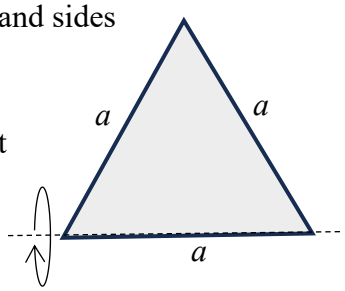
   I set this one up as an integral of differential elements of thin rods of length $2y$ rotated about their middles. The moment of inertia of a rod will be $dI = \frac{1}{12} dm(2y)^2$.

   The variable of integration is x.



   A. What is the mass dm of the thin rod of length $2y$, thickness dx, and area density σ?
   B. What are the limits of integration?
   C. What is the formula for y at a given value of x?
   D. What is the moment of inertia? Find a formula in terms of M and a.

6. What is the moment of inertia of an equilateral triangle of mass M and sides of length a when rotated about an edge?



I can think of two ways to set up dI. They could be thin rods perpendicular to the axis of length y and thickness dx pivoted about their ends, or they could be rods parallel to the axis of length $2x$ and thickness dy at a distance y from the axis. Both ways should work.

7. If you still have time, here's a surprisingly challenging problem: find the moment of inertia of an equilateral triangle of mass M and sides of length a rotated about an axis through its center of mass that is perpendicular to the plane of the triangle. I have evaluated this one by several different methods. Some of them are simpler than others, but I've made mistakes galore in all of them. I've consistently eventually gotten the same correct answer, but usually only after extensive proofreading. The best way I've found doesn't use integration at all; only geometry. It cleverly exploits the high symmetry of the problem.