PHYS 1210 Discussion Worksheet 9 Brief solutions

Woman leaning against a wall

There are four forces acting on the woman.

F_N from the wall on her shoulders in the $-\hat{\imath}$ direction

Her weight, $W = -500 \text{ N } \hat{\jmath}$, on her center of gravity (effectively her center of mass)

Friction f from the ground on the bottom of her feet in the $+\hat{\imath}$ direction

The normal force from the ground F_G on the bottom of her feet in the $+\hat{\jmath}$ direction

Two of these forces are horizontal and two are vertical. The vertical forces must be equal and opposite, as must the vertical forces. Immediately we know $F_G = +500 \text{ N } \hat{\jmath}$, and though we don't know either f or F_N, we know that they are opposite.

To find those remaining forces, we will employ the knowledge that the net torque must be zero. To do the problem correctly, all torques must be evaluated about the same reference point. Any reference point is valid, though for convenience it usually is best to select a point that involves the fewest nonzero torques. The obvious choice here is where her feet touch the ground. With that reference point, the torques from f and F_G both have zero lever arm. Then we need to formulate the torques from only F_N and W.

$$\tau_W = \vec{r}_W \times \vec{W} = (1.10 \text{ m}, 60°) \times (500 \text{ N}, 270°) = -275 \text{ N·m}$$

$$\tau_N = (1.50 \text{ m}, 60°) \times (F_N, 180°) = +(1.50 \text{ m}) F_N \sin 60°$$

$$\tau_W + \tau_N = \vec{0}$$

$$(1.50 \text{ m}) F_N \sin 60° = 275 \text{ N·m}$$

$$F_N = 211.7 \text{ N}$$

The force of friction has the same magnitude in the opposite direction.

PHYS 1210 Discussion 9 Problem 2

2A $\dfrac{mr + 2m(r+2D)}{3m} = \dfrac{m(r+2r+4D)}{3m} = \dfrac{3r+4D}{3} = \boxed{r + \frac{4}{3}D}$

2B $I_1 = m\left(\frac{4}{3}D\right)^2 + 2m\left(\frac{2}{3}D\right)^2 = m\left[\frac{16}{9}D^2 + \frac{8}{9}D^2\right] = \frac{24}{9}mD^2 = \frac{8}{3}mD^2$

2C $I_2 = mr^2 + 2m(r+2D)^2 = mr^2 + 2m(r^2+4rD+4D^2)$
$= m(r^2 + 2r^2 + 8rD + 8D^2) = m(3r^2 + 8rD + 8D^2)$

2D $V_{cm} = \omega_1\left(r + \frac{4}{3}D\right)$

2E $\frac{1}{2}m(\omega_1 r)^2 + \frac{1}{2}(2m)\left[\omega_1(r+2D)\right]^2 \leftarrow$ might be good this way
$= \frac{1}{2}m\omega_1^2 r^2 + m\omega_1^2(r^2 + 4rD + 4D^2)$
$= \frac{1}{2}m\omega_1^2 r^2 + m\omega_1^2 r^2 + 4m\omega_1^2 rD + 4m\omega_1^2 D^2$
$= \frac{3}{2}m\omega_1^2 r^2 + 4m\omega_1^2(rD + D^2)$
$= \left[\frac{3}{2}r^2 + 4D(r+D)\right]m\omega_1^2$

2F $\omega_1\left(r + \frac{4}{3}D\right)$ same as part D

2G same as part E

2H same $\omega_2 = \omega_1$

2I $3mD^2$

2J $\omega_1 I_1 = \omega_3 I_3$
$\omega_3 = \omega_1 I_1 / I_3 = \omega_1 \dfrac{8/3\, mD^2}{3mD^2} = \frac{8}{9}\omega_1$

2K Well $K_3 = \frac{1}{2}I_3\omega_3 = \frac{1}{2}(3mD^2)\left(\frac{8}{9}\omega_1\right)^2 + \frac{1}{2}MV_{cm}^2$
$= \frac{1}{2}\cdot 3 \cdot \frac{64}{81} mD^2\omega_1^2 + \frac{1}{2}MV_{cm}^2$
$= \frac{32}{27}mD^2\omega_1^2 + \frac{1}{2}MV_{cm}^2$

Initially, same $\frac{1}{2}MV_{cm}^2 + \frac{1}{2}\left(\frac{8}{3}mD^2\right)\omega_1^2$
$= \frac{4}{3}mD^2\omega_1^2$

How does $\frac{4}{3}$ compare to $\frac{32}{27}$? $\frac{4}{3} = \frac{36}{27}$, so it's bigger