PHYS 1210 Discussion 13. Fluids and Gravity

1. The basic design of any hydraulic system is two connected fluid-filled cylinders of different cross-sectional area. Each cylinder is fitted with a close-fitting piston. Ideally, the fluid is incompressible and the container does not expand, so that when one piston is compressed, it expels a volume of fluid from its cylinder directly into the other cylinder, pushing its piston.

   Your task is to design a hydraulic system. It must be able to lift a load of 3.0 metric tons using an input force of no more than 100 N. The pressure of the hydraulic fluid in the system must not exceed 10.0 bar during this lift.

   A. What is the minimum diameter of the output piston?

   B. What is the maximum diameter of the input piston?

   C. How would these numbers change if:

      i. the system needed to lift a heavier load, with the same considerations of pressure and input force?

      ii. the pressure of the hydraulic fluid could be higher?

      iii. the input force could be greater?

2. In lab, when you squeezed the 2-L bottle holding the floating test-tube diver, the test tube sank. What made it sink?

   To fully answer this question, you need to explain why the diver floated in the first place, and what changed to make it sink. Free body diagrams would be helpful, both for when the diver floated and when it sank.

3. Air pressure at sea level is 101,325 pascals. The Earth's radius is 6378 km.

   A. What is the weight of Earth's atmosphere?

   B. What is the mass of Earth's atmosphere? Assume the Earth's gravitational field is uniform.

   C. You now know that Earth's gravity decreases with distance from the surface, so the answer you just got must be wrong. Is it too low, or too high? Explain.

   D. If the air in Earth's atmosphere had a constant density of 1.204 kg/m^3 (its density at 20° C and 1 atmosphere pressure), how thick would the atmosphere be?

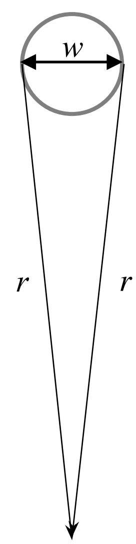
4. Have you heard that an object falling into a black hole becomes "spaghettified" as tidal forces pull it apart toward the attractor and crush it laterally? Let's see what Newton's gravitational formula says about such a situation. We won't worry about whether or not the object is a black hole; we'll just consider the formula for the gravitational field around a point mass.

First, the stretching part. We'd like to know how the gravitational field g changes with distance from the mass-M attractor $g = GM/r^2$ by finding its derivative with respect to r. This is the degree to which the gravitational field changes with distance.

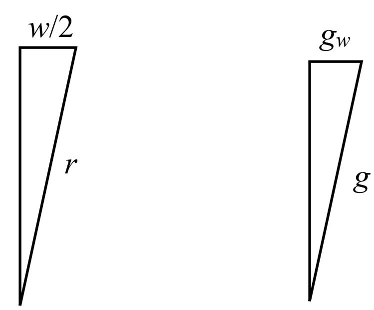
A. Find the formula for dg/dr.

B. The Earth's mass is 5.97×10^{24} kg. At what distance from a point with the mass of Earth does the gravitational field change by 1000 times earth's gravitational field every meter? That would be $dg/dr = 9800$ (N/kg)/m. I think that would be uncomfortable to experience.

What would the compressive lateral forces be at this distance? We don't need calculus for this; geometry is sufficient. Consider an isosceles triangle whose base is some width w and whose height is the distance r to the attractor. Gravity pulls down and inward along both edges of the triangle—what is the sideways component of that pull?



Specifically, we want to know the sideways compressional field (in the direction of w illustrated) per unit length of w. A little thought tells us that the two triangles sketched below are similar. The specific ratio we want to find is g_w/w.



C. What is the compressive lateral field g_w at the distance r where the tidal gradient dg/dr is 9800 (N/kg)/m? Would this be sufficient to "spaghettify" an object falling toward this attractor?