PHYS 1210 Discussion 14. Binary Systems

Two stars, one of mass m_1 and the other of mass m_2, are a distance *D* apart. They each circle their mutual center of mass. The only force acting on them is their mutual gravitational attraction.

1. It is convenient to label the distance from Star 1 to the center of mass of the binary system as r_1. Express r_1 in terms of m_1, m_2, and *D*.
2. Likewise, we can call the distance form Star 2 to the center of mass of the binary system r_2. Express r_2 in terms of m_1, m_2, and *D*.

The formulas for r_1 and r_2 should interconvert if all the indices are swapped. It should also be true that $r_1 + r_2 = D$.

3. The radius of Star 1's orbit is r_1. What is v_1, the tangential speed of Star 1 in its orbit? (It may be most convenient to solve for v_1^2 first.) Express in terms of *G*, m_1, m_2, and *D*.
4. The radius of Star 2's orbit is r_2. What is v_2, the tangential speed of Star 2 in its orbit? Express in terms of *G*, m_1, m_2, and *D*.
5. What is the total kinetic energy of the two orbiting stars?
6. What is the gravitational potential energy of the two orbiting stars?

You should notice a simple relationship between kinetic energy and potential energy. This relationship was also present in the simple limit of $m_1 \gg m_2$.

7. What is the angular momentum of the two stars about their center of mass?
8. The stars have different orbital radii and different orbital velocities, but they should always be directly opposite each other in their orbits. This means that they should have the same angular speed ω. Find ω, and express it in terms of *G*, m_1, m_2, and *D*. (It will probably be most convenient to solve for ω^2 first.) Remember that $\omega = v/r$.
9. With orbits, we usually are more interested in the orbital period than in either the tangential speed or angular speed. The orbital period is the time of one revolution. What is the orbital period of this binary system? Express it in terms of *G*, m_1, m_2, and *D*.
10. Let's look at a realistic example. Suppose $m_1 = 4 \times 10^{30}$ kilograms, $m_2 = 2 \times 10^{30}$ kilograms, and $D = 3 \times 10^{12}$ meters. What is their orbital period?

The reason this example is realistic is that these are close to the mass and orbital values of Sirius A and B, a star system close to ours. Sirius A is currently the brightest star in the sky. Sirius A is an A-type main sequence star, and Sirius B is a white dwarf. The only feature of the example that isn't realistic is that their orbits are substantially elliptical with an eccentricity of 0.59, not circular.