PHYS 1210 Spring 2026 Exam 3

Brief Solutions

1. Three forces on a wheel

The force in line with the center of the wheel has zero lever arm, so it applies zero torque. The other two forces apply torques in opposite directions: the tangential force in the positive (counterclockwise, out of the page) direction, and the force outward 40 degrees away from the radial direction in the negative (clockwise, into the page) direction.

A. Net torque

To find the net torque, we need to find the individual torque vectors $\vec{r} \times \vec{F}$ and sum them.

We know that the radial force produces zero torque.

The tangential force produces a torque of $+(0.350\text{ m})(8.50\text{ N}) = +2.975\text{ N}\cdot\text{m}$.

The oblique force produces a torque of $(0.350\text{ m})(14.6\text{ N})\sin(-40°) = -3.285\text{ N}\cdot\text{m}$.

These torques sum to $-0.310\text{ N}\cdot\text{m}$. The magnitude of the torque is $0.310\text{ N}\cdot\text{m}$.

B. Direction of the net torque

The net torque is negative or clockwise, which means that the torque vector is into the page. To figure that out using the right hand rule, curl the fingers of your right hand in the clockwise direction and extend that hand's thumb. It points into the page.

C. Equilibrant force magnitude

Exam version A asks for the oblique force magnitude necessary to produce a zero net torque. Thus

$$rF_2\sin(-40°) + rF_1 = 0$$
$$F_2 = \frac{-F_1}{\sin(-40°)} = \frac{-8.50\text{ N}}{-\sin(40°)} = 13.2\text{ N}$$

Exam version B asks for the tangential force magnitude necessary to produce a zero net torque. Thus

$$rF_2\sin(-40°) + rF_1 = 0$$
$$F_1 = -F_2\sin(-40°) = F_2\sin(40°) = 9.38\text{ N}$$

2. Playground Merry-go-round

The merry-go-round has a radius of 2.30 meters and a moment of inertia of $2400\,\text{kg}\cdot\text{m}^2$. In version A, the child pushes with a force of 20.0 newtons for 23.0 seconds; in version B, the child pushes with a force of 30.0 newtons for 17.0 seconds. In both cases, the force is tangential, so the resulting torque's magnitude is the force magnitude times the radius.

A. Final angular speed

We enter this kinematic problem by way of Newton's second law: $\alpha = \tau/I = rF/I$ finds the acceleration, then kinematics finds the final angular velocity. From acceleration, time, and initial angular velocity, the applicable kinematic formula to use to find final angular velocity is $\omega = \omega_0 + \alpha\Delta t$.

Version A: $\alpha = (2.30\,\text{m})(20.0\,\text{N})/(2400\,\text{kg}\cdot\text{m}^2) = 0.01917\,\text{rad/s}^2$, so $\omega = 0 + (0.01917\,\text{rad/s}^2)(23\,\text{s}) = 0.441\,\text{rad/s}$.

Version B: $\alpha = (2.30\,\text{m})(30.0\,\text{N})/(2400\,\text{kg}\cdot\text{m}^2) = 0.02875\,\text{rad/s}^2$, so $\omega = 0 + (0.02875\,\text{rad/s}^2)(17\,\text{s}) = 0.489\,\text{rad/s}$.

B. Angle traversed

The kinematic formulas that can be applied here are $\Delta\theta = \omega_0 t + 1/2\,\alpha t^2 = 1/2\,\alpha t^2$, or $\Delta\theta = 1/2\,(\omega+\omega_0)t$. Both formulas come out to $1/2\,\alpha t^2$.

Version A gives 5.07 radians; Version B gives 4.15 radians.

C. Work done

You can approach this as torque times angle traversed, $W = \tau\Delta\theta$, or as force times distance traversed, $W = F\Delta\theta$: either way, the formula is $W = Fr\Delta\theta$.

Version A gives 233 joules; Version B gives 287 joules.

3. Collapsing star

This scenario doesn't match the process that forms a neutron star in all particulars. The collapsing stars initially are much more massive than the Sun, they expel much of their mass into Space rather than entirely collapsing, and the collapsing portion needs to be at least 1.4 times the Sun's mass. And, as the problem points out, the initial star does not have a uniform mass distribution. But accounting for those details would just make the problem more complex without changing the core ideas.

The core idea in this problem is conservation of angular momentum. The angular momentum $I\omega$ is the same before and after the collapse, although the star's moment of inertia I changes drastically as its radius shrinks.

A. Initial moment of inertia

We're told to model the star as a uniform sphere. The moment of inertia of a uniform sphere is $2/5\,MR^2$, where M is its mass and R its radius.

In version A, the mass is $M = 2.0\times10^{30}\,\text{kg}$ and radius $R = 7.0\times10^5\,\text{km} = 7.0\times10^8\,\text{m}$. This gives $I = 3.92\times10^{47}\,\text{kg}\cdot\text{m}^2$.

In version B, the mass is $M = 3.0\times10^{30}\,\text{kg}$ and radius $R = 8.0\times10^5\,\text{km} = 8.0\times10^8\,\text{m}$. This gives $I = 7.68\times10^{47}\,\text{kg}\cdot\text{m}^2$.

B. Final moment of inertia

After collapse, we still treat the star as a uniform sphere, so the formula for its moment of inertia is still $2/5\,MR^2$. Its mass is the same, but its radius has changed.

In version A, the final radius is 9000 m, giving a moment of inertia of $6.48\times10^{37}\,\text{kg}\cdot\text{m}^2$.

In version B, the final radius is 10,000 m, giving a moment of inertia of $1.20\times10^{38}\,\text{kg}\cdot\text{m}^2$.

C. Initial rotational kinetic energy

The rotational kinetic energy of an object with moment of inertia I spinning with angular speed ω is $1/2\,I\omega^2$. We just calculated the moment of inertia for the star, but we haven't been given the initial angular speed ω_0. We must find it from the period T, which is 34 days = 2,937,600 seconds. Accordingly, $\omega_0 = 2\pi/T = 2.14\times10^{-6}\,\text{rad/s}$.

In version A, $I = 3.92\times10^{47}\,\text{kg}\cdot\text{m}^2$, so $K = 8.97\times10^{35}\,\text{J}$.

In version B, $I = 7.68\times10^{47}\,\text{kg}\cdot\text{m}^2$, so $K = 1.76\times10^{36}\,\text{J}$.

D. Initial angular momentum

Angular momentum about the center of mass of a spinning object is $L = I\omega$. We have already found the moment of inertia I and the angular velocity ω.

For version A, the result is $L = 8.38 \times 10^{41}\,\mathrm{kg \cdot m^2}$.

For version B, the result is $L = 1.64 \times 10^{42}\,\mathrm{kg \cdot m^2}$.

E. Final angular momentum

Here's the key point: angular momentum is conserved, because no external forces are exerting a torque on the star. Consequently, the angular momentum after the collapse is the same as it was before the collapse.

F. Is initial moment of inertia overestimated or underestimated?

With the mass distribution closer to the center, the moment of inertia won't be as large as if the mass distribution were uniform from the center to the edge. So the moment of inertia calculated in part A is too large.

G. Is initial angular momentum overestimated or underestimated?

The calculated moment of inertia I was too large, and the calculated angular momentum is $I\omega$, so the calculated angular momentum is also too large.

4. Simple Harmonic Motion

The question gives the value of the oscillator's mass m, and the amplitude A and frequency f of the oscillation.

The frequency f alone allows us to find the angular frequency $\omega = 2\pi f$ and period $T = 1/f$ of the oscillation. The spring constant k and mass m together determine the angular frequency ω, as $\omega^2 = k/m$.

A. Period

$T = 1/f$. For Version A, that's $1/(2.25\,\mathrm{cycle/s}) = 0.444\,\mathrm{s/cycle}$. For version B, that's $1/(2.45\,\mathrm{cycle/s}) = 0.408\,\mathrm{s/cycle}$.

B. Angular frequency

$\omega = 2\pi f$. For version A, that's $(2\pi\,\mathrm{rad/cycle})(2.25\,\mathrm{cycle/s}) = 14.14\,\mathrm{rad/s}$. For version B, that's $(2\pi\,\mathrm{rad/cycle})(2.45\,\mathrm{cycle/s}) = 15.39\,\mathrm{rad/s}$.

C. Spring constant

From $\omega^2 = k/m$, we obtain $k = m\omega^2$. For version A , that's $(0.400\,\mathrm{kg})(14.14\,\mathrm{rad/s})^2 = 79.9\,\mathrm{kg/s^2}$; for version B, it's $(0.450\,\mathrm{kg})(15.39\,\mathrm{rad/s})^2 = 106.7\,\mathrm{kg/s^2}$.

What about the units? The question asked for newtons per meter, and the formula gave us kilograms per square second. Well, newtons are $\mathrm{kg \cdot m/s^2}$, so dividing by meters gives us $\mathrm{kg/s^2}$.

D. Maximum speed

The simplest way to find maximum speed is $v_{\max} = A\omega$. For version A, that's $(2.0\,\mathrm{cm})(14.14\,\mathrm{rad/s}) = 28.27\,\mathrm{cm/s} = 0.283\,\mathrm{m/s}$. For version B, it's $(2.0\,\mathrm{cm})(15.39\,\mathrm{rad/s}) = 30.79\,\mathrm{cm/s} = 0.308\,\mathrm{m/s}$.

E. Total mechanical energy

Because energy switches between elastic potential energy of the spring and kinetic energy of the glider, the total kinetic energy is the maximum potential energy $U_{\max} = 1/2\,kA^2$, and also the maximum kinetic energy $K_{\max} = 1/2\,mv_{\max}^2$. In this problem, we found the spring constant from $k = m\omega^2$, and the maximum kinetic energy from $A\omega$, so that gives us $U_{\max} = 1/2\,m\omega^2 A^2$ and $K_{\max} = 1/2\,m(A\omega)^2$;

exactly the same thing.

For version A, that is 0.0160 joules; for version B, it is 0.0213 joules.

5. Simple pendulum

The important relation with a simple pendulum swinging through a small angle is $\omega^2 = g/L$, where $\omega = 2\pi/T_1$ is the angular frequency of the oscillation, g is the acceleration due to gravity, and L is the length of the pendulum.

A. Different mass

Changing the mass of the bob will have no effect on the period. There is no mass dependency in g/L.

B. Different length

Changing the length from L to $L_2 = aL$ changes $\omega_1^2 = g/L$ to $\omega_2^2 = g/L_2 = g/(aL) = (g/L)/a = \omega_1^2/a$. Period $T_2 = 2\pi/\omega_2 = 2\pi/(\omega_1/\sqrt{a}) = \sqrt{a}(2\pi/\omega_1) = \sqrt{a}\,T$.

In version A, $a = 2$, so $T_2 = \sqrt{2}\,T$. In version B, $a = 1/2$, so $T_2 = T/\sqrt{2}$.

C. Different gravity

Here we change the gravitational acceleration from Earth's $g_1 = 9.8\,\mathrm{m/s^2}$ to a second body's g_2, given as $g_2 = ag_1$. In version A, the second body is the Moon, with $a = 1/6$; in version B, the second body is Mars, with $a = 0.4$.

$$\begin{aligned} T_2 &= 2\pi/\omega_2 \\ &= 2\pi/\sqrt{g_2/L} \\ &= 2\pi\sqrt{L/g_2} \\ &= 2\pi\sqrt{\frac{L}{ag_1}} \\ &= \frac{1}{\sqrt{a}}2\pi\sqrt{L/g_1} \\ &= \frac{1}{\sqrt{a}}T \end{aligned}$$

For version A, $T_2 = T/\sqrt{a} = T/\sqrt{1/6} = \sqrt{6}\,T \approx 2.45\,T$. For version B, $T_2 = T/\sqrt{0.4} \approx 1.58\,T$. In both scenarios, the period is longer where the gravity is weaker, which makes sense.

6. Physical pendulum with square bob

In the small angle approximation, the physical pendulum is a Hooke's law torsional oscillator, which means that the torque follows angular Hooke's law $\tau = -\kappa\theta$ (restoring force is directly proportional to angular displacement), with $\kappa = mgL$. Math tells us that the solution to Newton's second law $\alpha = \tau/I$ with this torque is a sinusoid $\theta = \Theta\sin(\omega t + \phi)$, where Θ depends on the total mechanical energy and $\omega^2 = \kappa/I$. We know $\kappa = mgL$; the parallel axis theorem tells us that $I = I_{\mathrm{cm}} + mL^2$. For this square bob, $I_{\mathrm{cm}} = 1/12\,m(a^2 + a^2) = 1/6\,ma^2$. Thus

$$\omega^2 = \frac{\kappa}{I} = \frac{mgL}{1/6\,ma^2 + mL^2} = \frac{gL}{a^2/6 + L^2}.$$

In version A, $m = 1.20\,\mathrm{kg}$, $a = 12.0\,\mathrm{cm}$, and $L = 9.0\,\mathrm{cm}$, so $\omega^2 = 84\,\mathrm{rad^2/s^2}$, thus $\omega = 9.165\,\mathrm{rad/s}$ and

$f = \omega/(2\pi) = 1.46\,\text{Hz}$.

In version B, $m = 1.20\,\text{kg}$, $a = 10.0\,\text{cm}$, and $L = 7.50\,\text{cm}$, so $\omega^2 = 100.8\,\text{rad}^2/\text{s}^2$, thus $\omega = 10.04\,\text{rad/s}$ and $f = \omega/(2\pi) = 1.60\,\text{Hz}$.

7. Type of wave

When the propagation of the wave is parallel to the distortion of the medium, the wave is longitudinal. When the propagation of the wave is perpendicular to the distortion of the medium, the wave is transverse.

8. Units of angular frequency

Angular frequency is expressed in radians per second. It tells the rate at which the phase of the oscillation or wave advances as time passes.

9. Angular wavenumber

Angular wavenumber tells how the phase of the wave advances with distance. Thus, it is related to the wavelength λ: the phase advances by 2π in distance λ. With a sinusoidal wave expressed as $A\sin(kx-\omega t+\phi)$, kx goes to $kx + 2\pi$ when x goes to $x + \lambda$.

$$\begin{aligned} k(x+\lambda) &= kx + 2\pi \\ kx + k\lambda &= kx + 2\pi \\ k\lambda &= 2\pi \\ k &= 2\pi/\lambda \end{aligned}$$

10. Formula description of a wave

Versions A and B had different parameter values and different signs for the time term. Both were of the form

$$y(x,t) = B\cos\left[2\pi\left(\frac{x}{L} \pm \frac{t}{\tau}\right)\right].$$

A. Amplitude

The amplitude is B in the formula, which for version A is 6.10 mm and for version B is 5.10 mm.

B. Wavelength

Wavelength is the smallest change in x that gives a phase change of 2π. Inspection of the formula finds that to be L. In version A, that is 27.0 cm; in version B, it is 25.0 cm.

C. Frequency

Frequency is the number of times per second that the phase advances by 2π. Inspection of the formula reveals that the phase advances by 2π every time t advances by τ; this happens $1/\tau$ times every second, so $f = 1/\tau$. For version A, this is $f_A = 25.0/\text{s} = 25.0\,\text{Hz}$; for version B, $f_B = 20.0/\text{s} = 20.0\,\text{Hz}$.

D. Propagation speed

By this point, we know a number of parameters that can help us find v. The simplest to remember is $v = L/\tau$. From the standard wave formula we can use $v = \omega/k = (2\pi/\tau)/(2\pi/L)$. From the requirement that x advances to keep the phase constant, we have

$$2\pi x/L \pm 2\pi t/\tau = C$$
$$x/L = \mp t/\tau + \frac{C}{2\pi}$$
$$x = \mp\frac{Lt}{\tau} + \frac{LC}{2\pi}$$
$$v = dx/dt = \mp L/\tau$$

All of these give the same formula. The speed must be non-negative, that's just $v = L/\tau$. For version A, that gives 675 cm/s; for version B, that gives 500 cm/s.

E. Propagation direction

We see above the ungainly formula $\mp L/\tau$; that was there because the formula in version A used $x/L-t/\tau$ while the formula in version B used $x/L+t/\tau$. *Decreasing* the phase with time (version A) means that at a point in space, the wave moves forward ($+x$ direction), while *increasing* the phase with time (version B) means that at a point in space, the wave moves backward ($-x$ direction). The phase velocity derivation in part D (above) tells the same story.

11. String tension

The formula for the speed of a transverse wave in a flexible cord with mass per unit length μ under tension F is $v = \sqrt{F/\mu}$. In this question we are asked to find tension F, but we aren't given μ *or* v. Instead, we are given mass m, length L, frequency f, and wavelength λ.

Our strategy is to find $\mu = m/L$ and $v = \lambda f$, and from those find F.

$$v^2 = F/\mu$$
$$F = \mu v^2$$
$$F = \frac{m(\lambda f)^2}{L}$$

For Version A, $v = (0.800\,\text{m/cycle})(35.0\,\text{cycle/s}) = 28.0\,\text{m/s}$. Thus

$$F = mv^2/L = (0.150\,\text{kg})(28.0\,\text{m/s})^2/(2.80\,\text{m}) = 42.0\,\text{N}.$$

For version B, $v = (0.700\,\text{m/cycle})(35.0\,\text{cycle/s}) = 24.5\,\text{m/s}$. Thus

$$F = mv^2/L = (0.150\,\text{kg})(24.5\,\text{m/s})^2/(2.80\,\text{m}) = 32.2\,\text{N}.$$

12. Sound intensity

The question asks how sound intensity, which is power per unit area, must vary to change the decibel rating.

The relationship between a decibel rating β and intensity I is $\beta = (10\,\text{dB})\log_{10}(I/I_0)$, where I_0 is a standard reference intensity.

In this question, we are told only that the decibel rating must increase by a certain value $\Delta\beta$, which is 7.00 dB for version A and 3.00 dB for version B. We are also told the distance from the source, which is 20.0 meters in both versions. We know that intensity is inversely proportional to the square of the distance from the source, but the question doesn't specify any change in distance from the source. What are we to do?

All we can really deduce from the question is that the power output changes, changing the intensity and the decibel level in turn. So at power P_1, the decibel level is β_1, and at power $P_2 = aP_1$, the decibel level is $\beta_2 = \beta_1 + \Delta\beta$. Our task is to find the factor a. From these modest beginnings we can crack this problem.

We can use the decibel formula to relate a change in decibel level $\Delta\beta$ to a change in sound intensity.

$$\begin{aligned}\Delta\beta &= \beta_2 - \beta_1 \\ &= (10\,\text{dB})\log_{10}(I_2/I_0) - (10\,\text{dB})\log_{10}(I_1/I_0) \\ &= (10\,\text{dB})\left[\log_{10}(I_2/I_0) - \log_{10}(I_1/I_0)\right] \\ &= (10\,\text{dB})\left[\log_{10}\left(\frac{I_2/I_0}{I_1/I_0}\right)\right] \\ &= (10\,\text{dB})\log_{10}(I_2/I_1)\end{aligned}$$

The decibel level formula is given in terms of intensity, but we're asked to find a change in power, not in intensity. We ought to be able to do what is asked of us if we quantify the relationship between power and intensity. It is

$$I = \frac{P}{4\pi r^2},$$

where r is the distance from the source. This means that $I_1 = P_1/(4\pi r^2)$ and $I_2 = P_2/(4\pi r^2)$; the power P is different for the two cases, but the distance r is the same, 20.0 meters. Above, we need to find I_2/I_1; we find

$$\frac{I_2}{I_1} = \frac{P_2/(4\pi r^2)}{P_1/(4\pi r^2)} = \frac{P_2}{P_1},$$

which makes intuitive sense.

Then we have

$$\Delta\beta = (10\,\text{dB})\log_{10}(I_2/I_1) = (10\,\text{dB})\log_{10}(P_2/P_1)$$

which we proceed to solve for P_2/P_1.

$$\begin{aligned}\Delta\beta &= (10\,\text{dB})\log_{10}(P_2/P_1) \\ \frac{\Delta\beta}{10\,\text{dB}} &= \log_{10}(P_2/P_1) \\ P_2/P_1 &= 10^{\Delta\beta/(10\,\text{dB})}\end{aligned}$$

For version A, $\Delta\beta$ is 7.00 dB, so $P_2/P_1 = 10^{0.700} = 5.01$; for version B, $\Delta\beta$ is 3.00 dB, so $P_2/P_1 = 10^{0.300} = 2.00$.

13. Doppler shift

In both versions, the police car, which is the source of the sound, travels east at speed $v_S = 40.0\,\text{m/s}$, the source frequency is $1.20 \times 10^3\,\text{Hz}$, the detecting car travels west at speed $v_D = 20.0\,\text{m/s}$, and the speed of sound is $v = 342\,\text{m/s}$. The difference is that in version A, the source and detector are each moving toward the other, and in version B, the source and detector are each moving away from the other. This means that in version A, $v_S = +40.0\,\text{m/s}$ and $v_D = -20.0\,\text{m/s}$, while in version B, $v_S = -40.0\,\text{m/s}$ and $v_D = +20.0\,\text{m/s}$.

The Doppler shift formula I derived in class, consistent with the sign convention used above, is

$$f_D = f_S\frac{v - v_D}{v - v_S}.$$

Version A:

$$f_D = (1200\,\text{Hz})\frac{342 + 20}{342 - 40} = 1438\,\text{Hz} = 1.44 \times 10^3\,\text{Hz}.$$

Version B:

$$f_D = (1200\,\text{Hz})\frac{342 - 20}{342 + 40} = 1012\,\text{Hz} = 1.01 \times 10^3\,\text{Hz}.$$