PHYS 1210 Standards 1–13 Retest 1

Brief Solutions

Standard 1 Retest 1

1. Unit of surface area

   $\mathrm{kg}^0 \cdot \mathrm{m}^2 \cdot \mathrm{s}^0$

2. Unit of coefficient of friction

   $\mathrm{kg}^0 \cdot \mathrm{m}^0 \cdot \mathrm{s}^0$ A coefficient of friction is a pure ratio, thus unitless.

3. Unit of momentum·acceleration

   $(\mathrm{kg} \cdot \mathrm{m/s})(\mathrm{m/s}^2) = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^3 = \mathrm{kg}^1 \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-3}$

   This happens to be a watt.

4 Avoirdupois ounces in one Troy ounce

For clarity, I'll use "Toz" for a Troy ounce, "Aoz" for an Avoirdupois ounce, "Tlb" for Troy pound, and "Alb" for Avoirdupois pound.

$$\mathrm{Toz} \cdot \frac{\mathrm{Tlb}}{12\ \mathrm{Toz}} \cdot \frac{373.2417216\ \mathrm{g}}{\mathrm{Tlb}} \cdot \frac{\mathrm{kg}}{1000\ \mathrm{g}} \cdot \frac{\mathrm{Alb}}{0.45359237\ \mathrm{kg}} \cdot \frac{16\ \mathrm{Aoz}}{\mathrm{Alb}} = 1.097\ \mathrm{Aoz}$$

So a Troy ounce is more than an Avoirdupois ounce, but an Avoirdupois pound is more than a Troy pound because it has more ounces.

5. Hectares in a square kilometer

$$(1\ \mathrm{km})^2 \cdot \left(\frac{1000\ \mathrm{m}}{\mathrm{km}}\right)^2 \cdot \frac{\mathrm{ha}}{(100\ \mathrm{m})^2} = \mathrm{km}^2 \cdot \frac{10^6\ \mathrm{m}^2}{\mathrm{km}^2} \cdot \frac{\mathrm{ha}}{10^4\ \mathrm{m}^2} = 100\ \mathrm{ha}$$

6. Toise ("toises?" I don't know) in 5 fathoms

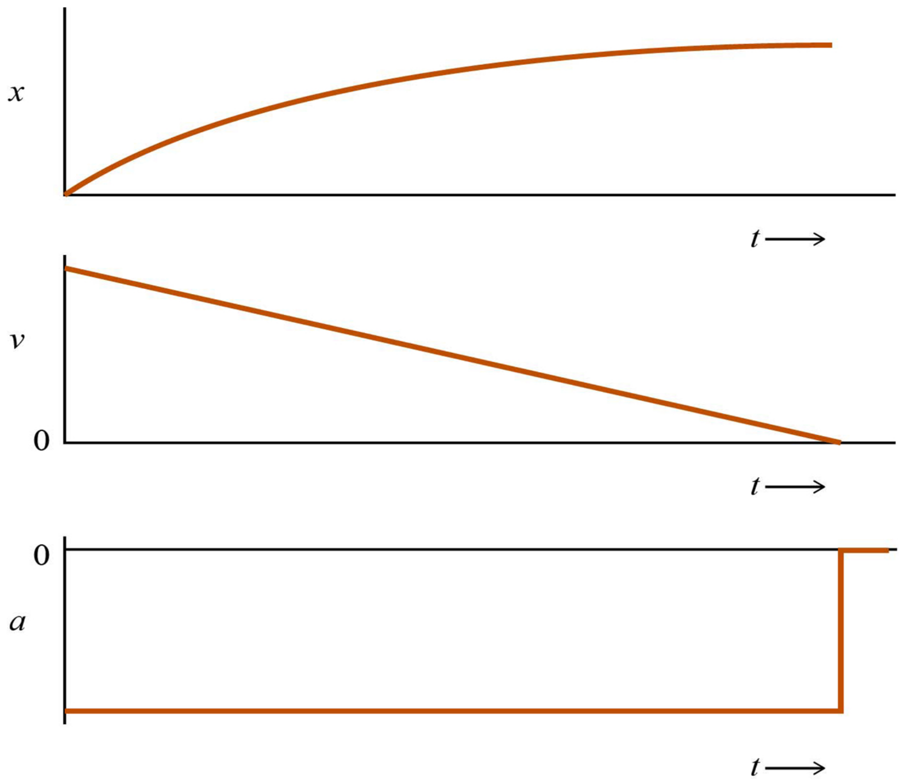
$$5\ \mathrm{fathom} \cdot \frac{6\ \mathrm{ft}}{\mathrm{fathom}} \cdot \frac{12\mathrm{in}}{\mathrm{ft}} \cdot \frac{2.54\ \mathrm{cm}}{\mathrm{in}} \cdot \frac{10\ \mathrm{mm}}{\mathrm{cm}} \cdot \frac{\mathrm{toise}}{1949.03631\ \mathrm{mm}} = 4.69\ \mathrm{toise}$$

So a toise is a little bit longer than a fathom.

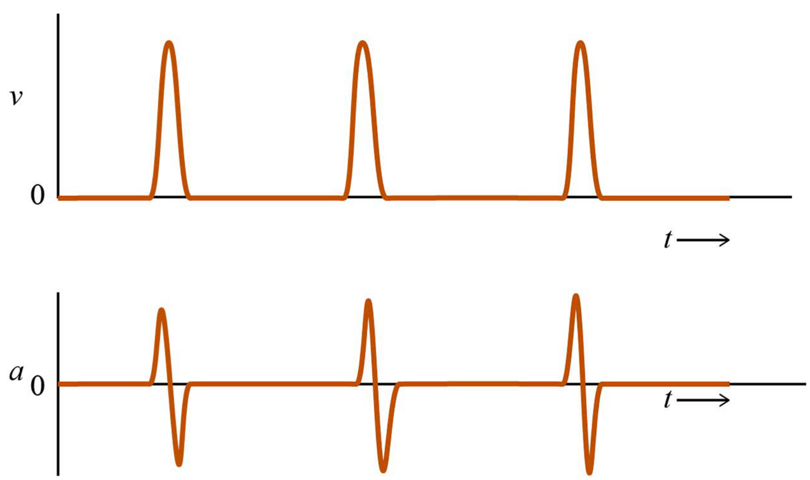
Standard 2 Retest 1

This retest was on paper during Exam 1 rather than online with the other retests.

1. Car slowing to a stop



2. Position-time graph given



These graphs show an object moving forward briefly and then stopping and resting three times. An example of such motion would be a student stepping away from a motion detector in three steps, pausing for a time between each step.

Standard 3 Retest 1

1. Average velocity

   Average velocity is change in position divided by duration, which is (3.3 km)/(12.5 min), which we then need to convert to meters per second. 1 km = 1000 m, and 1 min = 60 s.

2. Relationships between position, velocity, and acceleration

   Acceleration is the rate of change of velocity, which is the rate of change of position.

3. Zero acceleration and speed

   Acceleration is the rate of change of *velocity*, so if acceleration is zero, velocity is constant. Velocity encompasses both speed and direction, so a constant velocity must mean a constant speed.

Standard 4 Retest 1

1. Meaning of negative velocity

   Velocity is dx/dt, or, on average, $\Delta x/\Delta t$. This quantity is negative if x decreases as t increases.

2. Hare speed to beat the Tortoise

   The hare needs to reach the finish line before the tortoise. We are given the tortoise's speed v_T, the distance the tortoise needs to travel to reach the finish x_T, and the hare's distance to the tortoise D. The tortoise will finish in time $t = x_T/v_T$, so that hare needs to finish at least that soon. Thus he needs to travel a distance $x_T + D$ in time x_T/v_T, at a speed at least $v_T(x_T + D)/x_T$.

3. Mango Monkey

   This problem involves constant acceleration. It concerns two mangoes with the same acceleration of $-g$, but different initial speeds and different start times, arriving at the ground at the same time. We can attack it by first finding the time t_1 for the first mango to land on the ground. That is also the time that the second mango needs to land.

   We are given the time delay t_d between the first mango starting to fall and the monkey throwing the second mango downward. That means that the second mango reaches the ground in time $t_2 = t_1 - t_d$ after the monkey throws it. Both mangoes follow the general kinematic formulas $v = v_0 - gt$ and $y = y_0 - v_{0y}t - 1/2\ gt^2$. (Making the speed v_0 positive but the initial velocity negative.) Both mangoes have initial height $y_0 = H$. The first mango has initial speed 0, and the second has initial speed v_2, which is the unknown quantity we are trying to find. To find the time t_1, we solve $y = H - 1/2\ gt^2$ for t when $y = 0$.

$$
\begin{aligned}
y_1 &= H - 1/2\ gt^2 \\
0 &= H - 1/2\ gt_1^2 \\
t_1 &= \sqrt{2H/g}
\end{aligned}
$$

   Then we solve the equation for the second mango landing on the ground in time t_2 for v_2 when $y = 0$.

$$
\begin{aligned}
y_2 &= H - v_2 t - 1/2\ gt^2 \\
0 &= H - v_2 t_2 - 1/2\ gt_2^2 \\
v_2 t_2 &= H - 1/2\ gt_2^2 \\
v_2 &= H/t_2 - 1/2\ gt_2
\end{aligned}
$$

Standard 5 Retest 1

1. Graphical vector addition

   Place the tail of vector $\vec{B}$ at the head of vector $\vec{A}$. The resultant vector $\vec{C}$ starts at the tail of vector $\vec{A}$ and ends at the hesd of vector $\vec{B}$.

2. Direction θ of a vector given components (x, y)

   We know $\tan\theta = y/x$, so we start with $\arctan(y/x)$. If $x < 0$, we must add 180° to the result.

3. Magnitude R of a vector given components (x, y)

   $R = \sqrt{x^2 + y^2}$.

4. Adding vectors given as components $\vec{A} = (A_x, A_y)$ and $\vec{B} = (B_x, B_y)$

   $\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$

   I had to check these answers by hand, because Canvas considers it a fill-in-the blanks question, not a numerical question. Unfortunately, there isn't a course standard for following directions. "One decimal place" means "19.0," not "19." Sigh.

5. x component of vector $\vec{w}$ in coordinates rotated by angle $-\theta$ (clockwise)

   In these coordinates, $+x$ is downward and to the right, so the x compomnent of $\vec{w}$ is positive. $w_x = +w\sin\theta$.

6. y component of vector $\vec{w}$ in coordinates rotated by angle $-\theta$

   In these coordinates, $+y$ is upward and to the right, so the y component of $\vec{w}$ is negative. $w_y = -w\cos\theta$.

Standard 6 Retest 1

1. Quantities depending on g

   Those quantities are the ones with g in their formulas, which are the vertical quantities and not the horizontal quantities. I had to hand-grade this problem because the way Canvas grades multiple-answer problems doesn't match how many of the answers you got correct.

2. Launcher range

   You found and used this formula for the second lab. You use the horizontal shot information to find the muzzle speed v_0 of the projectile, and then use that muzzle speed to find the horizontal range from a launch at a different angle. Symbols below are initial height = h, initial speed = v_0, launch angle = θ, and D is the horizontal range from the lhorizontal ($\theta = 0$) shot.

   For the horizontal shot,

$$y = h - 1/2\ gt^2$$
$$0 = h - 1/2\ gt^2$$
$$t^2 = 2h/g$$
$$t = \sqrt{2h/g}$$

$$v_0 = D/t$$

   Then for a shot from arbitrary θ, assuming that v_0 does not change,

$$v_{0y} = v_0 \sin\theta;\ \ v_{0x} = v_0 \cos\theta$$
$$y = h + v_{0y}t - 1/2\ gt^2$$

$$0 = h + v_{0y}t - 1/2\ gt^2$$
$$0 = t^2 - 2\frac{v_{0y}}{g}t - \frac{2h}{g}$$
$$t = \frac{v_{0y}}{g} + \sqrt{\left(\frac{v_{0y}}{g}\right)^2 + \frac{2h}{g}}$$
$$x = v_{0y}t$$

Standard 7 Retest 1

1. Rotation period T given radius r and speed v

   $v = 2\pi r/T$, so $T = 2\pi r/v$.

2. Circumference given radius

   $2\pi r$

3. Period T given radius r and acceleration a

   $a = 4\pi^2 r/T^2$, so $T = 2\pi\sqrt{r/a}$. To get a numerical answer in SI units, you need to convert a given in g's to m/s^2.

Standard 8 Retest 1

Net force on a body traveling at constant velocity

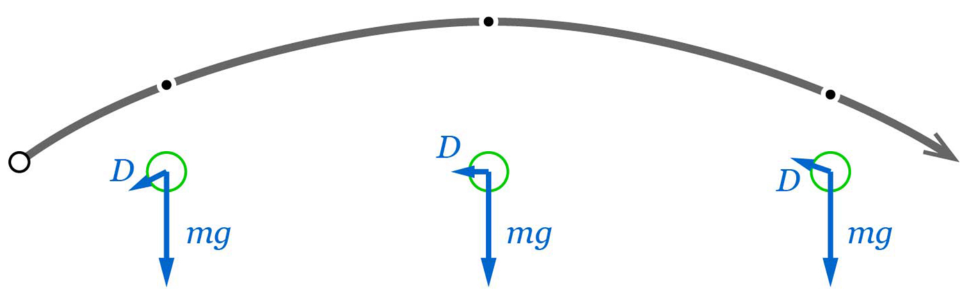
Constant velocity means zero acceleration, which means zero net force.

Standard 9 Retest 1

Three free body diagrams. It's a shame that the pictures didn't all load, but at least there were descriptions.

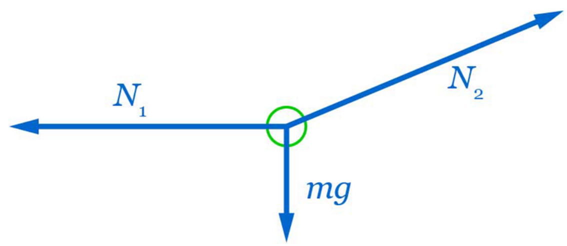
1. Golf ball in flight

   The question didn't specify which part of the trajectory to model, whether the upward arc, or the top of the arc, or the downward arc. But in all cases the force of the ball's weight is the same, and downward. If you included the drag on the ball, it would be opposite the direction of the ball's velocity at that time.



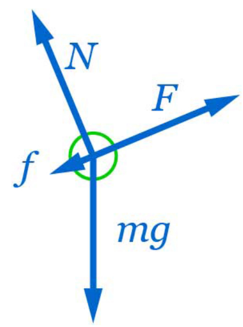
2. Ball wedged in a notch

   Here the forces acting on the ball are its downward weight and the normal forces from the two walls of the notch. The normal forces are perpendicular to the walls. Because this is a static situation, the forces add to zero. One normal force is wholly horizontal, so the other one must have a vertical component equal to the ball's weight. The horizontal normal force must be equal to the horizontal component of the other normal force.



3. Cart pulled by a cord

   If a friction force is included in the free body diagram (optional, as the cart has wheels), it should point either uphill or downhill; the question didn't specify which direction the cart was rolling. In any event, if the wheels roll well, friction will be a small contribution to the net force.



Standard 10 Retest 1

1. Formula for a horizontal tension

   The three tensions in the diagram add to zero. The vertical tension F_1 is equal to the weight of the sphere. The diagonal tension F_3 must have a vertical component also equal to the weight of the sphere. The horizontal tension F_2 must be equal to the horizontal component of the diagonal tension F_3. The trigonometric ratios are

$$\sin\theta = F_2/F_3$$
$$\cos\theta = F_1/F_3$$
$$\tan\theta = F_2/F_1$$

   We are asked to find F_2 in terms of F_1. For that, we would use the tangent relationship, $F_2 = F_1 \tan\theta$.

2. Horizontal tension

   Unfortunately, the formula that you found in question 1 gives the wrong answer for question 2, because the given angles are different. The angle specified in question 2 is the complement of the angle given in question 1. In *this* diagram, $\sin\theta = F_1/F_3$, $\cos\theta = F_2/F_3$, and $\tan\theta = F_1/F_2$.

   The easiest way to solve this problem would be to use the formula from question 1, finding the angle needed for that formula from the complement of the angle given. For the θ given in the question, the formula to use is $F_2 = F_1/\tan\theta$.

Standard 11 Retest 1

Net force on a crate

> You are given three horizontal forces, and you can safely assume that the vertical forces cancel. Your task, then, is to find the sum of the three horizontal forces. You know the full vectors of the pushes, but you aren't given the direction of the friction force. To figure that out, you must remember that friction opposes the motion. Find the sum of the two pushes; the friction is in the opposite direction.

Standard 12 Retest 1

Acceleration of an Atwood Machine

> The net force on the left block is $m_1 a = T - m_1 g$, and the net force on the right block is $m_2 a = m_2 g - T$. Both blocks have the same acceleration a, and both segments of the cable have tension T. The system of those two equations gives $a = g\ (m_2 - m_1)/(m_1 + m_2)$.
>
> In case you're curious, the system of equations also gives for the other unknown $T = 2g\ (m_1 m_2)/(m_1 + m_2)$. But the question didn't ask for that.

Standard 13 Retest 1

1. Weight given free-fall acceleration

   Acceleration $a = \Sigma F/m$, where ΣF is the pie's weight. Thus the weight is $\Sigma F = ma$.

2. Friction on a sliding object

   Kinetic friction $f = \mu N$; here on a level floor, $N = mg$, so $f = \mu mg$.

3. Direction of static friction force

   The static friction force is whatever it needs to be to keep the box from sliding. Since the box is at rest on a sloped surface, the force of static friction is directed uphill.

4. Magnitude of static friction

   The static friction is whatever it needs to be to keep the box from sliding. Its magnitude here is $mg\sin\theta$, cancelling the parallel component of the box's weight, the only other force with a nonzero parallel component. The formula $f_s \le \mu_s N$ does not give the magnitude of static friction directly; it only tells the maximum possible magnitude.

5. Normal force on a box on a slope

   The normal force is whatever it needs to be to keep the box from accelerating perpendicular to the slope. That magnitude is $mg\cos\theta$, cancelling the component of the box's weight in that direction.

6. Component of weight parallel to incline

   $mg\sin\theta$.

7. Component of weight perpendicular to incline

   $mg\cos\theta$.