Worksheet 3: Unit and Vector Algebra

Objectives

- Convert between units.
- Carry out mathematical operations with vectors.

Summary

Quantities and units

All the phenomena we will discuss in this class can be described in terms of three fundamental quantities: **distance**, **mass**, and **time**. The respective mks or SI units for these are the **meter** m, **kilogram** kg, and **second** s. The units for all physical quantities used in this course can be expressed in terms of these fundamental units.

Units of measurement can be operated on mathematically just like variables in algebra—they add together, subtract from each other, multiply, and divide. Converting between units of the same quantity requires identifying a proportional relationship between them.

Example: 1 h = 60 min, so 24 h = 24 (60 min) = 1440 min.

Vectors

Many physical quantities have particular directions and are expressed as **vectors**.

Often expressed as **components**: $\vec{A} = \left(A_x, A_y, A_z\right) = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$.

Unit vectors: $\hat{\imath}$ = (1, 0, 0); $\hat{\jmath}$ = (0, 1, 0); $\hat{k}$ = (0, 0, 1)

Magnitude: $\left\|\vec{A}\right\| = A = \sqrt{A_x + A_y + A_z}$

Addition: $\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y, A_z + B_z)$

Scalar multiplication: $c\vec{A} = c\left(A_x, A_y, A_z\right) = \left(cA_x, cA_y, cA_z\right)$

Scalar product (**dot product**): $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z = ABcos(\theta)$, where θ is the angle from $\vec{A}$ to $\vec{B}$.

Vector product (**cross product**): $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$; $\left\|\vec{A} \times \vec{B}\right\| = ABsin(\theta)$.

$\vec{A} \times \vec{B}$ is a vector, perpendicular to both $\vec{A}$ and $\vec{B}$, in the direction given by the right-hand rule.

Problems

1. What are the units of **volume**? Suppose another student tells you that a cylinder of radius r and height h has a volume given by $\pi r^3 h$. Explain why this cannot be right.

2. Light travels in a vacuum 2.9979×10^8 m in 1 s. How many nanoseconds does it take to travel 1.00 ft?

3. If the dot product of two vectors is negative ($\vec{A} \cdot \vec{B} < 0$), what does that tell you about them?

4. The magnitude of the cross product of two vectors is proportional to the sine of the angle between them. Vector magnitudes are always considered positive, yet a sine can be negative as well as positive. If $\sin(\theta)$ of the angle from $\vec{A}$ to $\vec{B}$ is negative:
   a. What does that mean about the angle θ?

   b. What does that mean for the direction of the vector $\vec{A} \times \vec{B}$?

5. A baseball thrown from the origin follows a path described by $x = 15\frac{\text{m}}{\text{s}}t$, $y = 15\frac{\text{m}}{\text{s}}t - 5\frac{\text{m}}{s^2}t^2$.
   a. Find the formulas for the horizontal (x-) components of velocity v_x and acceleration a_x.

   b. Find the formulas for the vertical (y-) components of velocity v_y and acceleration a_y.

c. Sketch graphs of x, v_x, and a_x with time.

d. Sketch graphs of y, v_y, and a_y with time.

e. Find the formula for its speed $v = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v_x^2 + v_y^2}$ with time.

f. Find the formula for the magnitude of its acceleration $a = \sqrt{\vec{a} \cdot \vec{a}} = \sqrt{a_x^2 + a_y^2}$ with time.

g. Find the formula for the rate of change of its speed dv/dt with time.