Worksheet 4: Uniform Gravity Kinematics

Objective

Work with ballistic trajectories in the vertical and horizontal directions.

Summary

Projectiles

When the only force is gravity (no air resistance, etc.), the horizontal (x) and vertical (y) components of the motion can be considered independently. For a projectile launched from (x_0, y_0) with initial speed v_0 at angle θ above horizontal, the initial velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath} = v_0 \cos\theta\,\hat{\imath} + v_0 \sin\theta\,\hat{\jmath}$ and

$x = x_0 + v_{0x}t$ $\quad$ $v_x = v_{0x}$ $\quad$ $a_x = 0$

$y = y_0 + v_{0y}t - {}^1/_2\, gt^2$ $\quad$ $v_y = v_{0y} - gt$ $\quad$ $a_y = -g$

(These require that the $+y$ direction be up.)

Range Equation

The horizontal distance traveled by a projectile fired at speed v_0 and angle θ and landing at its launch height is $v_0^2 \sin(2\theta)/g$.

Problems

1. A projectile is launched from height y_0 at speed v_0 and angle θ above the horizontal.
   a. Find the formula for the time at which the projectile reaches the top of its arc. Find the formula for the maximum height reached by the projectile.

   b. Find the formula for the maximum height reached by the projectile.

c. Find the formula for the horizontal distance the projectile travels to the top of its arc.

d. Find the formula for the time that the projectile lands on the ground.

e. Find the formula for the horizontal distance traveled from launch to landing on the ground.

f. Find the formula for the time taken by the projectile to travel a horizontal distance *L*.

g. Find the formula for the height of the projectile at horizontal distance L.

2. A museum trebuchet launches a 30.0-kg boulder catapult at a speed of 65.0 m/s at an angle of 50° above horizontal. The boulder is released 7.0 m above the ground.

   a. Where (horizontal displacement from the launch and height above the ground) is the top of its arc?

   b. What is the boulder's speed at the top of its arc?

   c. Where does the boulder land?

   d. What is the speed of the boulder when it lands?

   e. What is the incoming angle of the boulder's trajectory when it lands, as an angle below horizontal?