PHYS 1210 Worksheet.: Simple Harmonic Motion

Simple Harmonic Motion

Hooke's law (ideal spring): $F = -kx$;

Work to stretch or compress an ideal spring from rest to displacement x: $W = \frac{1}{2} kx^2$

Governing relationship: $-kx = m\frac{d^2x}{dt^2}$

Problems

1. Consider a mass m oscillating subject to the force of a Hooke's law spring with spring constant k.
   A. How should the oscillation frequency depend on m?

   B. How should the oscillation frequency depend on k?

2. Let us speculate that the position of the mass follows a formula
   $$x = A\sin(\omega t + \varphi)$$
   where A, ω, and φ are constants.
   A. If $x(t = 0) = A$, what is φ?

   B. If $x(t = 0) = -A$, what is φ?

   C. If $x(t = 0) = 0$, what is φ?

   D. What is the velocity of the mass, $v = dx/dt$?

   E. What is the acceleration of the mass, $a = d^2x/dt^2$?

F. Can that expression for x as a function of t satisfy the governing equation

$$m\,d^2x/dt^2 = -kx?$$

If so, under what conditions?

G. What is the formula for the net force at time t?

H. What is the formula for the kinetic energy at time t?

I. What is the formula for the elastic potential energy at time t?

J. What is the formula for the total mechanical energy at time t?