LAB 10. TORQUE AND MOMENT OF INERTIA

Introduction

In this activity you will use a falling mass to pull a string, which will generate a torque on a pulley that accelerates a heavy rotor with an unknown moment of inertia. You will measure the acceleration of the rotor to determine the rotor's moment of inertia.

Supplies

Rotor apparatus, string, photogate "smart pulley", interface and computer with Logger Pro software installed, 100-g, 200-g, and 500-g hanging masses, Vernier calipers, ruler.

Apparatus

The rotor is mounted in the base with its axis vertical. The rotor shares its axle with three pulleys of different radii and a disk with ten slots cut into it. One end of a string is wound around one of the pulleys, so that pulling the string applies a torque about the axis. The string is placed over an external pulley and a mass is hung from the other end, so that the weight of the hanging mass pulls the string. An auxiliary mass can be placed on the rotor to increase its inertia. A photogate installed around the slotted disk measures its rotation.

Theory

As the hanging mass falls, it turns the axle of the rotor. When the rotor and pulley of radius r turn through an angle θ radians, the string and attached weight move a distance $y = \theta r$. Correspondingly, when the rotor turns at angular speed ω radians per second, the weight moves at speed $v = \omega r$. When the rotor's angular acceleration is α radians per second per second, the hanging weight accelerates with acceleration $a = \alpha r$.

The only force promoting the descent of the mass is its gravitational attraction mg to the earth. Opposing the descent is the tension T in the string suspending it. The rotation of the rotor is driven by the torque $\tau = rT$ applied by the string via the rotor pulley of radius r. The net force on the hanging mass is a downward $\Sigma F = mg - T$, and the only torque on the rotor is $\Sigma\tau = rT$. The acceleration of the hanging mass is then $a = \Sigma F/m$, and the angular acceleration of the rotor is $\alpha = \Sigma\tau/I$.

The photogate detects if its light path is open or blocked. By counting the number of times the photogate's open/blocked state alternates, Logger Pro calculates the angular distance θ traversed by the slotted disk, and the angular velocity $\omega = d\theta/dt$. It can find only the absolute value of changes in θ; it has no information about the direction of θ, or consequently of ω or α. You need to provide the direction from your knowledge of the situation.

Experiment

In this activity three different hanging masses will pull on the string, with the string wound around the three different pulleys on the rotor axle. In each of the nine cases, you will measure the acceleration of the rotor at least twice, for a minimum of 18 trials.

Data Collection

Setup

1. Measure the mass and diameter of the rotor.
2. Use the Vernier calipers to measure the diameters of each of the three pulleys on the rotor axle.
3. Install the photogate around the slotted disk, connect the photogate and interface to the lab laptop, and recognize the photogate in Logger Pro. Download and open the experiment file "Rotary Photogate". It will calculate and plot θ and ω of the rotor from the photogate input.
4. Mount the rotor onto the axle.

Measurements

1. Wind the string around one of the pulleys, leaving enough free to hang the pulling weight. Check that the string winds around the pulley you want.
2. Hold the rotor to keep it from turning. Place the string over the photogate pulley and hang a mass at the end of the string.
3. Start data collection. Release the rotor to allow it and the hanging weight to move.
4. Just before the falling mass reaches the floor, stop the rotor. (If the rotor keeps spinning, the string may get tangled around the axis.) Stop data collection.
5. Fit the linear portion of the ω-t plot with a linear trend line. The slope of this trend line is the angular acceleration α of the rotor.
6. Record the hanging mass, the pulley radius, the fitted acceleration (trend line slope), and the uncertainty u (±) of the acceleration.
7. Repeat each run. If the two accelerations are not within 5% of each other, measure a third time.
8. Carry out the measurements using each of the three masses with each of the three pulleys on the rotor.

Extension

9. If time permits, repeat with the auxiliary ring mass added to the rotor.

Work-Up

Theory

Solve the equations in the Theory section above to find a formula for angular acceleration in terms of m, r, and I.

Data Processing

1. Make a spreadsheet that collates your data in a table. Each row of the table should be a single run. In the columns, enter all the measured quantities for the runs: r, m, and α.
2. In another column in the spreadsheet, calculate the predicted angular acceleration α_c using the formula you derived. You may also want to make columns for intermediate calculations, or for the estimate of I from each run.

3. What? You need to know the rotor's moment of inertia *I* to calculate its acceleration? Interesting. What can you do about that? You *can* estimate *I* from each measurement, by solving the equations of motion for *I*. But then you have an *I* for each run, and they will inevitably be different from each other. But there is only one rotor, and it has only one *I* about its axis. So how do you find the one overall best estimate of *I*?

   Oh, I have an idea! The point of this lab is to figure out what the rotor's inertia is. Perhaps you can make a guess of *I* and calculate what all the accelerations would be with that *I*. You can test how well the predicted accelerations a_c match the measured accelerations. Then, you can try out different values of *I* to find the one that makes the predictions come out closest to the measurements.

   Of course, doing that means that you need some way to quantify "how well the predicted accelerations a_c match the measured accelerations." For a single acceleration, it can just be the "residual" difference $a_{ci} - a_i$. Is there a way you can summarize all the residuals in an overall statistic for the whole data set? How about the sum of squares of residuals $S = \sum_1^N (\alpha_{ci} - \alpha_i)^2$? Set up the spreadsheet to calculate the "fit penalty" score *S*. Then find the *I* value giving the least *S*.
4. You will need to keep track of the *I* values you have tried, and their corresponding penalty scores. You can't know that you have the optimal value without other values to compare it to.
5. An optimal *I* parameter giving the best overall fit is useful, but it doesn't tell you if the model actually fits the data well. Find a way to scale the fit score to show how close a typical calculated acceleration is to the measured value.
6. Independently, calculate what the moment of inertia of the rotor should be by approximating it as a uniform solid cylinder, knowing its mass and radius.

Lab Report guidance

For this lab, make a standard lab report with the customary sections. Also upload or provide a link to your spreadsheet. Below are some considerations specific to this particular lab.

Abstract

What is the system? What quantities did you measure directly? What quantities do you infer from your measurements?

Purpose

This activity involves finding an unknown quantity without directly calculating it from a formula. What skills does this promote?

Theory

Derive a model for the angular acceleration α based on the characteristics of the system *r*, *m*, and *I*. Also provide the theoretical basis for evaluating the goodness of fit of an estimate of the rotor's moment of inertia *I*. Explain all of this.

Experimental

What was your experimental apparatus? How did you take measurements? What steps did you take to minimize measurement errors?

Observation and Data

Your primary data should be in your notebook, but you also need to transcribe them to your spreadsheet.

Analysis and Discussion

How did you assign your optimal estimate of *I*? How did you evaluate the goodness of fit? What factors might affect the acceleration that are not accounted for in the model? Do you have evidence that any of these might be detectable, or even substantial?

Is your experimental estimate of the moment of inertia close to the value calculated assuming it to be a uniform solid cylinder?

Conclusion

What is your best estimate of the moment of inertia of the rotor? Does the kinematic model you developed for the system adequately describe the accelerations?