PHYS 1220 Exam 1

Brief Solutions

1. Tennis ball packing gas

The gas is nitrogen, which is diatomic (the "2" in N_2), so its internal energy is given by $U = 5/2\ nRT$. When it is heated at constant volume, it does zero pV work, so the heat released equals its decrease in internal energy,

$$Q = \Delta U = 5/2\ nR\Delta T = 5/2\ \frac{p_1 V_1}{T_1}\Delta T = 5/2\ \frac{(1.50 \times 10^4\ \text{Pa})(0.200 \times 10^{-3}\ \text{m}^3)}{297.15\ \text{K}}(-29\ \text{K}) = -7.32\ \text{J}$$

The question asks for the heat *released*, so that is a positive number, 7.32 joules.

2. Argon in a balloon

In this problem, the gas is heated at a constant *pressure*, so some of the input heat energy goes into pushing the surroundings away rather than increasing the internal energy. The gas is monatomic (There is no subscript in the formula "Ar"), so its internal energy is $U = 3/2\ nRT$; the work done by the gas in pushing away the surroundings is $W = p\Delta V$. The heat input needs to account for both:

$$Q = \Delta U + W = 3/2\ nR\Delta T + p\frac{nR}{p}\Delta T = 5/2\ nR\Delta T = 5/2\ \frac{pV_1}{T_1}$$

$$\Delta T = 2/5\ Q\ \frac{T_1}{pV_1} = (0.4)(16.0\ \text{J})\ \frac{291.15\ \text{K}}{(9.00 \times 10^4\ \text{Pa})(1.70 \times 10^{-3}\ \text{m}^3)} = 12.18\ \text{K}$$

$$T = T_1 + \Delta T = 18.0^\circ\text{C} + 12.18^\circ\text{C} = 30.18^\circ\text{C}$$

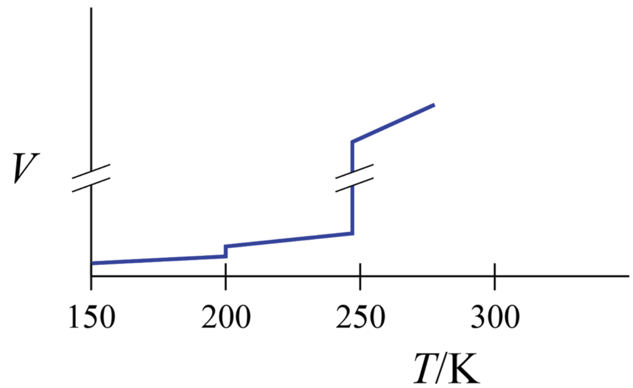
3. Phase diagram for ammonia

The path shown goes from about 150 K to about 275 K at a constant pressure of a little over 100,000 Pa. It begins in the"solid" region, passes into "liquid" at around 200 K, and then into "gas" at around 250 K.

A. Volume

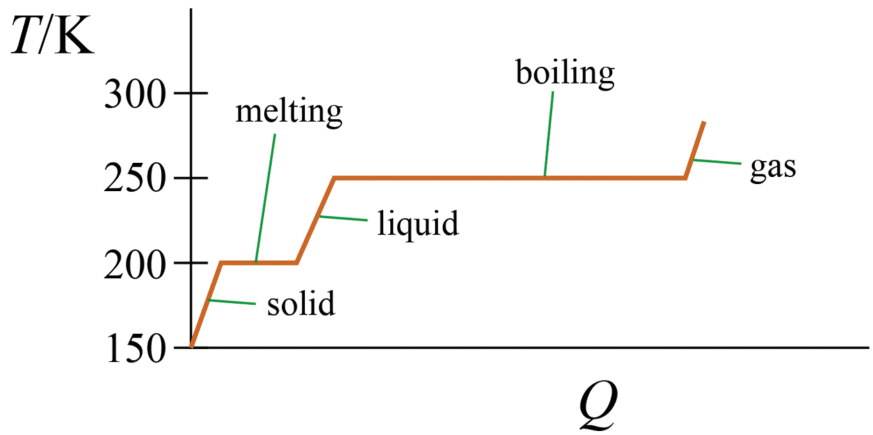
The volume will increase slightly with temperature in the solid and liquid phases, and the gas volume will be proportional to (absolute) temperature. This also means that the curve in the gas region will line up with zero volume at absolute zero temperature.

We also expect an increase in volume transitioning from solid to liquid (ammonia is not water), and a larger increase in volume transitioning from liquid to gas. The scale of the volume increase transitioning to gas will be greatly attenuated at high pressures, but at our pressure of around 1 atmosphere, the increase will be huge.



B. Heating curve

Temperature will increase with heat input by $\Delta T = Q/(mc_p)$ within a phase, and it will hold steady at phase boundaries until the latent heat of the phase transition has been input.



4. Steam engine heating step

A. Work done by the water

Although the boiling water is not an ideal gas, for any constant-pressure process $W = p\Delta V$. We know the pressure and the initial and final volumes.

$$W = p\Delta V = (10^6 \text{ Pa})(19.40 \text{ m}^3 - 0.113 \text{ m}^3) = 19.287 \times 10^6 \text{ J}$$

B. Change in internal energy

The first law of thermodynamics tells us $\Delta U = Q - W$. We know that the heat added is $Q = 201.4 \times 10^6$ J, and we just calculated the work done.

$$\Delta U = 201.4 \times 10^6 \text{ J} - 19.287 \times 10^6 \text{ J} = 182.1 \times 10^6 \text{ J}.$$

C. Entropy change

The process is a phase change; at constant pressure it happens at a specific temperature. So we have a particular case of constant pressure and constant temperature but *not* constant volume. Because temperature is constant, we can use $\Delta S = Q/T$ to calculate the entropy change.

$$Q/T = (201.4 \times 10^6 \text{ J})/(453.15 \text{ K}) = 444 \times 10^3 \text{ J/K}.$$

D. Entropy change with half as much working fluid

If there were half the fluid, it would absorb half as much heat, and the entropy change would be half as much. That's choice b.

5. Diesel compression stroke

We are given the initial pressure, temperature, and volume; and also the final volume. The compression ratio $V_1/V_2 = 18$. The working fluid is a diatomic ideal gas.

I will not derive the formulas for adiabatic compression of an ideal gas here; that was done in lecture. Here I'll assume that you have the formulas.

A. Final temperature

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{2/f} = (300\text{ K})(18)^{0.4} = 953.3\text{ K}$$

B. Final pressure

The classic formula for an adiabatic volume change of an ideal gas is $p_1 V_1^\gamma = p_2 V_2^\gamma$, where γ is the specific heat capacity ratio $c_p/c_v = (f+2)/f$.

$$p_2 = p_1 (V_1/V_2)^{7/5} = (8.9 \times 10^4\text{ Pa})(18)^{1.4} = 5.09 \times 10^6\text{ Pa}$$

C. Work done

In an adiabatic process, $Q = 0$, so $W = Q - \Delta U = -\Delta U$. We find ΔU from the change in temperature.

$$\Delta U = f/2\; nR\Delta T = f/2\; \frac{p_1 V_1}{T_1}(T_2 - T_1) = 5/2$$

6. Multiplicity ratio

We are given the entropy change ΔS and asked for Ω_2/Ω_1. From the formula $\Delta S = k \ln(\Omega_2/\Omega_1)$, we obtain

$$\Delta S = k \ln(\Omega_2/\Omega_1)$$
$$\Delta S/k = \ln(\Omega_2/\Omega_1)$$
$$\exp(\Delta S/k) = \Omega_2/\Omega_1$$
$$\Omega_2/\Omega_1 = \exp\left(\frac{11.8\text{ J/K}}{1.380649 \times 10^{-23}\text{ J/K}}\right) = \exp(8.55 \times 10^{23})$$

This formula would overflow a calculator if you tried to evaluate it, so I'll stop there.

7. Thermodynamic temperature and heat flow

A. Absolute temperature?

This has to be absolute temperature. What would this formula mean at 0°C or 0°F? At 0 K, it means that any heat input gives an infinite increase in temperature, which truly is meaningful. But it doesn't make sense in a temperature scale that is not absolute.

B. Heat flow direction

The more entropy S increases from a process, the more likely the process is to occur. A body with a large dS/dU gains a large amount of entropy by absorbing heat; likewise, a body with a smaller dS/dU gains a smaller amount of entropy by absorbing the same amount of heat. Therefore, overall entropy increases when heat flows from a body with small dS/dU to a body with large dS/dU. (I think of dS/dU as a hunger for heat.) Since temperature is the reciprocal of dS/dU, a high hunger for heat means a low temperature, and heat transfer from a high-temperature body to a low-temperature body increases entropy overall.

8. Diffusion of dice

There is only one way to put all dice in a single zone: each die must be in that zone. There are many ways to distribute dice evenly among the zones, however.