PHYS 1220 Exam 2

Brief Solutions

1. Proposed heat engines

Six heat engines are presented, the only information about them being Q_h, W, and Q_c.

A. First law of thermodynamics

The first law requires $Q_h = W + Q_c$.

Engine	Q_h, W	W, W	Q_c, W	$W + Q_c$, W
A	1000	400	600	1000
B	1000	1000	0	1000
C	1000	600	600	1200
D	1000	500	500	1000
E	1000	450	550	1000
F	1000	650	200	850

From this table, we see that Engines C and F do not obey the first law of thermodynamics, and are therefore impossible.

B. Second law of thermodynamics

The second law requires entropy to increase overall. In a heat engine, the entropy of the hot reservoir decreases by an amount $\Delta S_h = -Q_h/Th$, and the entropy of the cold reservoir increases by an amount $\Delta S_c = +Q_c/T_c$. Entropy then increases overall if

$$-Q_h/Th + Q_c/T_c \geq 0$$
$$Q_c/T_c \geq Q_h/Th$$

Engine	Q_h, W	W, W	Q_c, W	Q_h/T_h, W/K	Q_c/T_c, W/K
A	1000	400	600	10/6	2
B	1000	1000	0	10/6	0
D	1000	500	500	2	5/3
E	1000	450	550	2	55/25

Of the remaining engines, B and D do not increase entropy overall, and thus are ruled out by the second law of thermodynamics.

C. Greatest efficiency

Efficiency of a heat engine is $e = W/Q_h$. We can compare our remaining contenders.

Engine	Q_h, W	W, W	e
A	1000	400	0.40
E	1000	450	0.45

E is the most efficient of the possible engines.

2. Refrigerators and heat pumps

A. Second law constraint

In refrigerators and heat pumps, entropy increases in the hot reservoir and decreases in the cold reservoir, $\Delta S_h = +Q_h/T_h$ and $\Delta S_c = -Q_c/T_c$. The constraint is

$$\begin{gathered} +Q_h/T_h - Q_c/T_c \geq 0 \\ +Q_h/T_h \geq Q_c/T_c \end{gathered}$$

B. Heat pump COP

The benefit divided by the cost is Q_h/W.

C. Refrigerator COP

The benefit divided by the cost is Q_c/W.

3. Statements about static charges

a. TRUE, electric charge is conserved.

b. TRUE, positive and negative charges attract.

c. TRUE, charge is quantized, existing only in specific amounts.

d. FALSE, positive charges repel; they do not attract.

e. FALSE, neutral objects contain cancelling positive and negative charges.

f. TRUE, negative charges repel.

4. Charges on the x and y axes

A. Force magnitude

Particle A is on the x axis at $3\,\mathrm{m}\,\hat{\imath}$ and particle B is on the y axis at $12\,\mathrm{m}\,\hat{\jmath}$. Coulomb's law reveals

$$F = k\frac{q_1 q_2}{r^2} = 8.987 \times 10^9\,\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}^2}\,\frac{(+700 \times 10^{-6}\,\mathrm{C})(-200 \times 10^{-6}\,\mathrm{C})}{153\,\mathrm{m}^2} = 8.22\,\mathrm{N}$$

B. Force direction

We don't need to calculate anything more about Coulomb's law; this is just a little trigonometry. The charges of the two particles have opposite signs, so they attract. Particle B is pulled toward particle A, and the force vector is in the same direction as the separation vector, which is $-3.0\,\mathrm{m}\,\hat{\imath} + 12.0\,\mathrm{m}\,\hat{\jmath}$. This vector is in quadrant 2; the arctangent function returns an angle in quadrant 1 or 4. The angle we want here is

$$\theta = 180^\circ + \arctan\left(\frac{12}{-3}\right) = 180^\circ + \arctan(-4) = 180^\circ - 75.96^\circ = 104^\circ$$

5. Statements about field lines and isopotential surfaces

a. TRUE. Feld lines and isopotential surfaces are perpendicular.

b. TRUE. Static charges rearrange in a conductor to create zero electric field, which means constant potential.

c. FALSE. Potential is constant on an isopotential surface.

d. FALSE. Potential is constant on an isopotential surface.

e. TRUE. "Isopotential" means equal potential.

6. Isopotential surfaces and electric field

A. Magnitude of electric field

Electric field is the negative gradient of potential. Where isopotential surfaces are close together, the potential changes the most over a short distance. Hence, the field is greatest there. The indicated point where the isopotential surfaces are closest together is D.

B. Field directions

Field lines point "downhill" on the potential surface.

At A, downhill is to the left and a little down.

At B, downhill is down and a little to the left.

At C, downhill is close to straight down.

At D, downhill is to the left.

7. Gauss's law

A. Charge enclosed

The charge depends on the length of the line of charge enclosed by the surface. In this case, the surface encloses $L = 0.40$ meters of the line, which bears a charge of $\lambda L = (6.40\,\mu\text{C/m})(0.400\,\text{m}) = 2.56\,\mu\text{C}$.

B. Electric flux

We have two possible ways to find electric flux: by integrating a known field over the surface, or from the charge enclosed by the surface. We don't know the field, but we do know the charge. The surface encloses a positive charge, so the flux through it will be positive.

$$\Phi = Q/\varepsilon_0 = \frac{2.56 \times 10^{-6}\,\text{C} \cdot \text{N} \cdot \text{m}^2}{8.854 \times 10^{-12}\,\text{C}^2} = 2.89 \times 10^5\,\text{N} \cdot \text{m}^2/\text{C}$$

C. Electric field

Here we invert the definition of flux. We know the field is the same magnitude at all points on this cylindrical shell, and that it is perpendicular to the shell, so $\Phi = EA$. Thus, to find the field E, we need only to divide the flux by the area of the shell, which is $2\pi r L$.

$$E = \Phi/A = \frac{2.89 \times 10^5\,\text{N} \cdot \text{m}^2/\text{C}}{2\pi(0.100\,\text{m})(0.400\,\text{m})} = 1.15 \times 10^6\,\text{N/C}$$

D. Flux if the Gaussian surface's radius were doubled

In that case, the surface would enclose the same charge, so the flux would be the same.

The area of the cylindrical shell would double, so the electric field would be half as strong at that distance from the line of charge. But the question asked about the enclosed charge, not about the electric field.

8. Dipole on the x axis

These questions all concern the electric potential at points around a dipole at the origin. Potential is the work required per charge moving to a particular location from an infinite distance away, where the potential is defined to be zero. The electric potential at a distance r from a single isolated point charge Q is kQ/r; if additional point charges are present, the potential is just the sum of the results from the individual source charges. In the case of this dipole, then, there will be two terms for the potential at the different field points: one for the positive source charge, and one for the negative source charge.

A. At field point $y = 0$, $x = +0.20\,\text{m}$

Here, $r_1 = 0.3\,\text{m}$, and $r_2 = 0.1\,\text{m}$.

$$\begin{aligned} V = V_1 + V_2 &= \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} = kQ\left(\frac{-1}{r_1} + \frac{1}{r_2}\right) \\ &= \left(8.987\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-9}\,\text{C}\right)\left(\frac{-1}{0.3\,\text{m}} + \frac{3}{0.3\,\text{m}}\right) \\ &= \left(15.00\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(\frac{2}{0.3\,\text{m}}\right) \\ &= 100.0\,\text{V} \end{aligned}$$

B. At the field point $y = 0$, $x = -0.20\,\text{m}$

Here, $r_1 = 0.1\,\text{m}$, and $r_2 = 0.3\,\text{m}$.

$$\begin{aligned} V = V_1 + V_2 &= \left(8.987\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-9}\,\text{C}\right)\left(\frac{-3}{0.3\,\text{m}} + \frac{1}{0.3\,\text{m}}\right) \\ &= \left(15.00\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(\frac{-2}{0.3\,\text{m}}\right) \\ &= -100.0\,\text{V} \end{aligned}$$

C. At the field point $y = 0$, $x = 0$

Here, $r_1 = 0.1\,\text{m}$, and $r_2 = 0.1\,\text{m}$.

$$\begin{aligned} V = V_1 + V_2 &= \left(8.987\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-9}\,\text{C}\right)\left(\frac{-1}{0.1\,\text{m}} + \frac{1}{0.1\,\text{m}}\right) \\ &= 0 \end{aligned}$$

9. 330-nF capacitor charged to 50 volts

A. Charge on the plates

$$Q = CV = (330\times10^{-9}\,\text{C/V})(50\,\text{V}) = 16.5\,\mu\text{C}$$

B. Energy

$$U = 1/2\,QV = 1/2\,CV^2 = 1/2\,(330\times10^{-9}\,\text{C/V})(50\,\text{V})^2 = 412.5\,\mu\text{J}$$

10. Combining a 330-nF capacitor with a 110-nF capacitor

In parallel, the equivalent capacitance is the sum of the individual capacitances. In series, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances. These questions ask about the charge $Q = CV$ on the 330-nF capacitor, so we need to understand how charge and voltage works in these configurations.

A. Charge on the 330-nF capacitor when in parallel

When in parallel, the two capacitors are each subjected to the same source voltage, which here is 50.0 volts. Thus the charge on the 330-nF capacitor is $(330 \times 10^{-9}\,\mathrm{C/V})(50.0\,\mathrm{V}) = 16.5\,\mu\mathrm{C}$.

B. Charge on the 330-nF capacitor when in series

The key to unlocking this question is to realize that the same charge is separated by both capacitors and by the equivalent capacitor. When charge Q is placed on the positive side of capacitor 1, $-Q$ is placed (or $+Q$ is removed) from the negative side of capacitor 2. To answer this question, then, we need to find the charge on the equivalent capacitor. Since $Q = CV$, we need to find the equivalent capacitance C and then multiply is by $V = 50.0$ volts.

$$\begin{aligned}
\frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{330\,\mathrm{nF}} + \frac{1}{110\,\mathrm{nF}} \\
&= \frac{1}{330\,\mathrm{nF}} + \frac{3}{330\,\mathrm{nF}} = \frac{4}{330\,\mathrm{nF}} \\
C &= \frac{330\,\mathrm{nF}}{4} = 82.5\,\mathrm{nF}.
\end{aligned}$$

$$Q = CV = (82.5\,\mathrm{nF})(50.0\,\mathrm{V}) = 4.125\,\mu\mathrm{C}$$

11. Comparing and adjusting parallel plate capacitors

Both capacitors have the same charge Q, but Capacitor 1 has plate area A and Capacitor 2 has plate area $2A$. We'll need to keep in mind the formula for capacitance of a parallel plate capacitor, $C = \kappa A \varepsilon_0 / d$, as well as the capacitor formulas $C = Q/V$ and $U = 1/2\,QV$.

A. Same plate spacing d: Find V_2 in terms of V_1

With the same plate spacing, Capacitor 2 will have twice the capacitance of Capacitor 1: $C_2 = 2C_1$. Voltages will be $V = Q/C$, so $V_1 = Q/C_1$ and $V_2 = Q/C_2 = Q/(2C_1) = 1/2\,(Q/C_1) = 1/2\,V_1$.

B. Spacing d_2 to make $C_2 = C_1$

$$\begin{aligned}
C_1 &= C_2 \\
\kappa A_1 \varepsilon_0 / d_1 &= \kappa A_2 \varepsilon_0 / d_2 \\
A_1/d_1 &= A_2/d_2 \\
A/d_1 &= 2A/d_2 \\
d_2 &= 2\,d_1
\end{aligned}$$

C. Dielectric κ to make $C_1 = C_2$

$$\begin{aligned}
C_1 &= C_2 \\
\kappa A \varepsilon_0 / d &= 2A\varepsilon_0/d \\
\kappa &= 2
\end{aligned}$$

12. Breakdown of a mica dielectric

The breakdown voltage V_B is determined by the dielectric strength E_D and the thickness d of the dielectric layer. The dielectric strength tells the highest electric field strength that the dielectric can withstand. In a parallel plate capacitor, the electric field E is uniform, so the voltage V, the work per charge necessary to push a test charge from the negative plate to the positive plate, is $V = E_D\,d$. Thus, the breakdown voltage of the dielectric layer will be $V_B = E_D\,d$.

A. Dielectric thickness d giving breakdown voltage $V_B = 600\,\mathrm{V}$

$$V_B = E_D\,d$$
$$d = V_B/E_D = (600\,\mathrm{V})/(110 \times 10^6\,\mathrm{V/m}) = 5.455 \times 10^{-6}\,\mathrm{m}$$

B. Plate area A giving capacitance $C = 5.00\,\mu\mathrm{F}$

$$C = \frac{\kappa A \varepsilon_0}{d}$$
$$A = \frac{Cd}{\kappa\varepsilon_0} = \frac{(5.00 \times 10^{-6}\,\mathrm{F})(5.455 \times 10^{-6}\,\mathrm{m})}{(4.0)(8.854 \times 10^{-12}\,\mathrm{F/m})} = 0.770\,\mathrm{m}^2$$

C. Electric field E inside the dielectric when $V = 300\,\mathrm{V}$

The field inside the dielectric is $1/\kappa$ what it would be if the dielectric were a vacuum. Here, that would be V/d, so

$$E = \frac{V}{\kappa d} = \frac{300\,\mathrm{V}}{4 \cdot 5.455 \times 10^{-6}\,\mathrm{m}} = 1.37 \times 10^7\,\mathrm{V/m} = 13.7\,\mathrm{kV/mm}$$