PHYS 1220 Exam 3

Brief Solutions

1. Resistor

For this problem, we need to use both $I = V/R$ and $P = VI$. We are given $P = 72\,\text{mW}$ and $V = 3.60\,\text{V}$.

A. Current I

We can use $I = P/V = (0.0072\,\text{W})/(3.60\,\text{V}) = 20\,\text{mA}$.

B. Resistance R

We could use $R = V/I$ using the I we just calculated, or we could sidestep propagating errors by using $R = V/(P/V) = V^2/P = (3.60\,\text{J/C})^2/(0.072\,\text{J/s}) = 180\,\text{J}\cdot\text{s/C}^2 = 180\,\text{V/A} = 180\,\Omega$.

2. Substitute aluminum for copper

Resistance is given by $R = \rho L/A$; here, cross-sectional area A is determined by diameter d by $A = \pi(d/2)^2 = \pi d^2/4$; resistivities ρ_c and ρ_a are given, and length L is unspecified, but is the same for the copper wire and aluminum wire. (You can also just make $L = 1\,\text{m}$.) For copper wire, $R_c = \rho_c L/A_c$; for aluminum wire, $R_a = \rho_a L/A_a$.

$$\begin{aligned}
R_c &= R_a \\
\rho_c L/A_c &= \rho_a L/A_a \\
\rho_c/(\pi d_c^2/4)c &= \rho_a/(\pi d_a^2/4) \\
d_a^2 &= d_c^2 \rho_a/\rho_c \\
d_a &= d_c\sqrt{\rho_a/\rho_c} \\
&= (1.63\,\text{mm})\sqrt{2.65/1.68} \\
&= 2.05\,\text{mm}
\end{aligned}$$

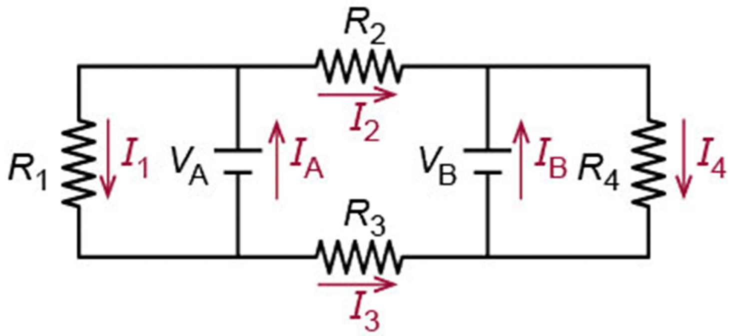
3. Resistive circuit

Step by step, we can reduce the resistor combinations to equivalent single resistors.

Resistors 1 and 2 in series combine to an equivalent resistor with resistance $R_A = R_1 + R_2 = 40\,\Omega$. Resistors 3 and 4 in series combine to an equivalent resistor with resistance $R_B = R_3 + R_4 = 120\,\Omega$. Those equivalent resistors in parallel combine to an equivalent resistor withe resistance R_C, where $1/R_C = 1/R_A + 1/R_B = 3/(120\,\Omega) + 1/(120\,\Omega) = 4/(120\,\Omega) = 1/(30\,\Omega)$, so $R_C = 30\,\Omega$. Then that resistor is in series with resistor 5, for an equivalent resistance of $R = R_C + R_5 = 45\,\Omega$. The current through this resistor, and thus through resistor 5, is $V/R = (9\,\text{V})/(45\,\Omega) = 0.20\,\text{A}$, thus $V_5 = I_5 R_5 = (0.20\,\text{A})(15\,\Omega) = 3\,\text{V}$.

4. Two-source circuit

It makes sense to show the currents through the voltage sources in the $\uparrow$ direction, and likewise through the resistors on the far left and far right (resistors 1 and 4, respectively) in the $\downarrow$ direction. For the other two resistors (2 and 3), the current directions will depend on the other parameters of the circuit. I'll arbitrarily set both of them to be positive in the $\rightarrow$ direction.



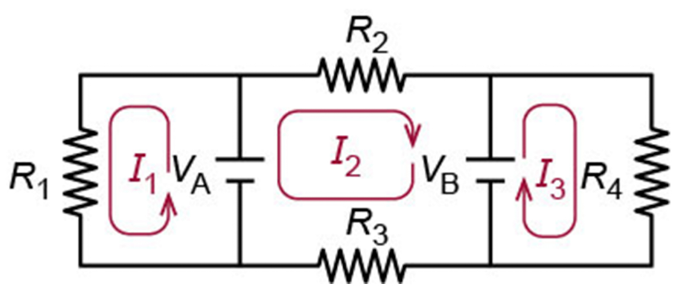
Voltage rule equations, left to right meshes:

$$0 = V_A - I_1 R_1$$
$$0 = V_A - I_2 R_2 - V_B + I_3 R_3$$
$$0 = V_B - I_4 R_4$$

Current rule equations, top left, top right, and bottom left nodes:

$$0 = -I_A + I_1 + I_2$$
$$0 = -I_B - I_2 + I_4$$
$$0 = I_A - I_! - I_3$$

Many students chose to analyze the circuit with mesh currents.



One advantage of this approach is that the complicated part of the problem only has three unknowns: the three mesh currents I_1, I_2, and I_3. Then the current through resistor 1 is I_1, the current through resistors 2 and 3 are both I_2, and the current through resistor 4 is I_3. But we're not finished: we still need to find the current through the voltage sources. Through source A, we have $I_A = I_1 + I_2$; through source B, we have $I_B = I_3 - I_2$. Here, there are five unknown currents rather than six because current I_2 applies to both resistors 2 and 3.

5. Discharging capacitor

The time constant of the circuit is $\tau = RC = 0.20\,\mathrm{s}$, and the equation for current is $I = I_0 e^{-t/\tau}$, where $I_0 = V_0/R = 0.500\,\mathrm{A}$. To find when the current is some particular I, solve for t.

$$I = I_0 e^{-t/\tau}$$
$$I/I_0 = e^{-t/\tau}$$
$$\ln(I/I_0) = -t/\tau$$
$$t = \tau \ln(I_0/I) = (0.20\,\mathrm{s}) \ln(10) = 0.46\,\mathrm{s}$$

6. Bar magnet in a dipole field

The magnet will rotate to align with the external magnetic field: its north end is pulled in the direction of the field, and its south end is pulled the opposite direction. It will also travel; the net force on the magnet is not zero in this non-uniform field. Once the magnet reorients at its initial position, its north end will be pulled more strongly than its south end, and the whole magnet will be pulled to the center of the field, where the field lines are most dense.

7. Energized solenoid interactions

Parallel currents attract; anti-parallel currents repel.

A. Adjacent coils

These currents are parallel, so adjacent coils attract. This will tend to keep the coils together.

B. Opposite segments of a single coil

These currents are antiparallel, so segments across a loop repel. This will tend to keep the loops open.

8. Solar wind proton

The proton will experience a Lorentz force from the magnetic field, given by $\vec{F} = q\vec{v} \times \vec{B}$. Directions in this problem are given in terms of the cardinal directions north, south, east, and west. These aren't Cartesian directions, but we can find the cross product in terms of what we're told.

A. Magnitude of force

The proton's speed is $v = 4 \times 10^5$ m/s; its east component is $v \cos 6°$ and its north component is $v \sin 6°$. The Earth's magnetic field here has magnitude $B = 4.80 \times 10^{-5}$ T; its east component is zero and its north component is B. This gives a force of $q\vec{v} \times \vec{B} = qvB \cos 6° = 3.06 \times 10^{-18}$ N in the upward direction. (The northward component of the velocity is inconsequential.)

B. Path of the proton

The proton will initially be deflected upward. Its speed will remain constant because the force is always perpendicular to its velocity. As its velocity becomes more upward and less eastward (its northward component never changes), the force gains a westward component. This causes the proton to circle in a plane perpendicular to the north-south axis, while moving northward with speed $v \sin 6°$. It follows a helical path.

What will be the radius of the helix? The tangential speed of the proton is $v_T = v \cos 6° = 398{,}000$ m/s and its axial speed is $v \sin 6° = 41{,}800$ m/s. The proton's centripetal force mv_T^2/r is the magnetic force qv_TB, giving

$$\begin{aligned} mv_T^2/r &= qv_TB \\ \frac{mv_T^2}{qv_TB} &= r \\ r &= \frac{mv_T}{qB} = 86\,\text{m} \end{aligned}$$

9. Loop next to a wire

In all of the scenarios in this problem, there will be an emf induced around the loop if and only if there is a magnetic flux change within the loop.

A. Loop motionless

There will be no flux change, so zero emf.

B. Loop moves to the left

The flux through the loop will not change, because the loop's distance from the wire will not change. Again, zero emf.

C. Loop moves to the right

As when the loop moved to the left, there is no flux change and thus zero emf.

D. Loop moves downward

The loop is moving to a region where the magnetic field is less intense. The field direction is out of the page ($\odot$). This induces a magnetic dipole in the loop to oppose the change, meaning that the dipole is also directed out of the page ($\odot$). The current direction achieving that is counterclockwise.

E. Loop moves outward ($\odot$)

Again, the loop moves to a region where the magnetic field is less intense. By the same reasoning, the current will be counterclockwise.

F. Loop motionless, current increases

This time, the magnetic flux through the loop *increases*. This will induce in the loop a magnetic dipole into the page away from you ($\otimes$). A clockwise current produces that dipole.

10. Loop magnetic moments

A. Greatest magnetic moment

The magnetic moment is proportional to the current around the loop and to the area of the loop. The loop giving the greatest magnetic moment is the one with the greatest area, which is the circle.

B. Magnetic moment direction

The direction of a current loop's magnetic moment is given by the right hand rule. A counterclockwise current will produce a magnetic moment out of the page, toward you ($\odot$).

11. Field of a toroid electromagnet

An appropriate Ampèrean loop is a circle in the plane of the toroid, centered about the toroid's principal axis, with a radius r between the inner and outer radius of the toroid. For this particular toroid, $4\,\text{cm} < r < 6\,\text{cm}$. From the symmetry of the toroid, we expect the B field to have the same magnitude everywhere along the loop. The current enclosed by the loop is NI; for this loop, $N = 50$ and $I = 40$ amperes. Ampère's law gives

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B\,2\pi r = NI$$

$$B = \frac{NI}{2\pi r}$$