

PHYS 1220-02 Fall 2026 Quiz 2

Brief Solutions

1. Heat to raise the temperature of a diatomic ideal gas at constant volume

The internal energy of the gas is given by $U = f/2 nRT$, so its change in internal energy is $\Delta U = f/2 nR\Delta T$ (n and R are constant). The first law of thermodynamics tells us that $\Delta U = Q - W$, and the constant volume tells us that the gas does no pV work, so $Q = \Delta U$. For a diatomic gas, the number of degrees of freedom f is 5. The last information we need to calculate the heat input is the ideal gas equation of state $pV = nRT$, which rearranges to $nR = pV/T$.

$$\begin{aligned} Q &= \Delta U \\ &= f/2 nR\Delta T \\ &= \frac{5}{2} \frac{p_1 V_1}{T_1} \Delta T \\ &= \frac{5}{2} \frac{(24 \times 10^5 \text{ Pa})(0.14 \text{ m}^3)}{293.15 \text{ K}} (30 \text{ K}) \\ &= \frac{5}{2} (1146.2 \text{ J/K})(30 \text{ K}) \\ &= 85,963 \text{ J} \end{aligned}$$

So the heat input is 86 kJ. That neglects the heat needed to raise the temperature of the cylinder itself; it's just the heat needed to raise the temperature of the contents.

2. Justify $C_p > C_v$

When heating a gas at constant volume, all that can happen is that the gas molecules gain kinetic energy. The added kinetic energy is supplied by the heat input. When heating a gas at constant pressure, it expands, doing work on its surroundings. The internal energy of the gas is reduced by the work done: $\Delta U = Q - W$. Thus, to obtain the same internal energy change, meaning the same temperature change, more heat must be added to compensate for the internal energy diverted to the work done.

I *meant* to write in the question statement to compare raising the temperature at constant *pressure* to raising the temperature at constant volume. My proofreading didn't catch that I wrote to raise the temperature at constant temperature. I think most of you grasped what I meant there, and I believe that Dilip wrote the correction on the board during the quiz.

3. Carbon dioxide phase diagram

A. State at 1 bar and -70°C

This is in the vapor region of the diagram, so the carbon dioxide is vapor (gas).

B. Constant-temperature pressurization starting at 1 bar and -70°C

i. Volume change

The volume will decrease. The volume of an ideal gas will be proportional to $1/p$.

ii. At the sublimation line

At the sublimation line, the gas transforms to the solid. If the temperature is maintained, the pressure does not increase until all the gas has deposited as the solid. On the p - V plot, the volume drops discontinuously. The material expels heat during that phase transition.

C. Pressure of liquid/vapor equilibrium at 20°C

The vapor/liquid equilibrium occurs at the saturation line. At 20°C, the pressure at the saturation line is 60 bars. The logarithmic plot is difficult to read, however, so a guess between 40 and 80 bars is defensible.

D. Disappearance of liquid at 40°C

According to the diagram, the critical point of carbon dioxide is at about 32°C and 75 bars. Above the critical temperature, there is no distinction between vapor and liquid: rather than an abrupt phase transition, the sample's volume changes continuously with pressure. Whether you want to call it a vapor, liquid, or supercritical fluid, only one phase is present.

4. Melting ice

A. Heat absorbed

For this, we use the latent heat of melting.

$$Q = mL = (0.500 \text{ kg})(334 \text{ kJ/kg}) = 167 \text{ kJ}$$

B. Work done

In a constant pressure process, the work W done by the system is $p\Delta V$. This is a constant pressure process in which the pressure is 10 bars. The volume change is negative, because the density of water ρ_W is greater than the density of ice ρ_I . This makes the work done negative as well.

Density $\rho = m/V$, so $V = m/\rho$. The initial state is ice and the final state is water, so $V_1 = m/\rho_I$ and $V_2 = m/\rho_W$, making

$$\Delta V = V_2 - V_1 = m/\rho_W - m/\rho_I = m \left(\frac{1}{\rho_W} - \frac{1}{\rho_I} \right).$$

The work done is then

$$\begin{aligned} W &= p\Delta V \\ &= pm \left(\frac{1}{\rho_W} - \frac{1}{\rho_I} \right) \\ &= (10^6 \text{ J/m}^3) (0.500 \text{ kg}) \left(\frac{1}{999.9 \text{ kg/m}^3} - \frac{1}{917 \text{ kg/m}^3} \right) \\ &= (10^6 \text{ J/m}^3) (0.500 \text{ kg}) (-9.04 \times 10^{-5} \text{ m}^3/\text{kg}) \\ &= 45.2 \text{ J} \end{aligned}$$

C. Internal energy change

We need to use the first law of thermodynamics $\Delta U = Q - W$. The work done, even though the pressure is well above atmospheric, is barely noticeable above the heat needed.

$$\Delta U = Q - W = 167,000 \text{ J} - 45.2 \text{ J} = 166,955 \text{ J} = 167 \text{ kJ}.$$

5. Adiabatic expansion

The gas is diatomic, so it has $f = 5$ degrees of freedom. Its initial temperature is $+30^\circ\text{C} = 303.15\text{ K}$, its final temperature is $-15^\circ\text{C} = 258.15\text{ K}$ and the initial pressure is $p_1 = 5.00\text{ bar} = 5.00 \times 10^5\text{ J/m}^3$.

A. Volume ratio V_2/V_1

I believe I derived in class a formula for volume and temperature. If you had that in your notes, or if you cared to derive it during the quiz, you could use it. But the catch-all formula most textbooks employ for adiabatic volume changes of gases is $pV^\gamma = \text{constant}$, where γ is the heat capacity ratio c_P/c_V . For an ideal gas, $\gamma = (f + 2)/f$. To get a formula involving volume and temperature from this, we can use the ideal gas equation $p = nRT/V$.

$$\begin{aligned} p_1 V_1^\gamma &= p_2 V_2^\gamma \\ \frac{nRT_1}{V_1} V_1^\gamma &= \frac{nRT_2}{V_2} V_2^\gamma \\ \frac{T_1}{V_1} V_1^\gamma &= \frac{T_2}{V_2} V_2^\gamma \\ T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-2} \\ \left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \frac{T_1}{T_2} \\ \frac{V_2}{V_1} &= \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \\ \frac{V_2}{V_1} &= \left(\frac{T_1}{T_2}\right)^{\frac{f}{2}} = \left(\frac{303.15}{258.15}\right)^{5/2} = 1.49 \end{aligned}$$

B. Work done

Since it is an adiabatic process, $Q = 0$ and $\Delta U = 0 - W$, and the work done is $W = -\Delta U$. We can find U and thus ΔU from the temperature. We were given the initial volume $V_1 = 10^{-3}\text{ m}^3$ for this part because the work depends on how much gas is expanding.

$$\begin{aligned} W &= -f/2 \ nR\Delta T \\ &= -\frac{f}{2} \frac{p_1 V_1}{T_1} \Delta T \\ &= -\frac{5}{2} \frac{(5.00 \times 10^5\text{ J/m}^3)(10^{-3}\text{ m}^3)}{303.15\text{ K}} (-45\text{ K}) \\ &= 186\text{ J} \end{aligned}$$

6. Poker hands

The royal flush is the most valuable poker hand because of its very low multiplicity. A four-of-a-kind also has a low multiplicity, but higher than the royal flush. Even if I had specified the most valuable four-of-a-kind, four aces, its multiplicity is 51.

A. Probability

The four-of-kind is more probable.

B. Entropy

The four-of-a-kind has higher entropy.

7. Irreversibility

Any significant entropy change represents a huge change in probability. It's not strictly impossible for an entropy-increasing change to reverse, but it is so improbable that there is no benefit to considering it.