PHYS 1220-02 Fall 2026 Quiz 4

Brief Solutions

1. Capacitor formulas

A. Charge, potential, and capacitance

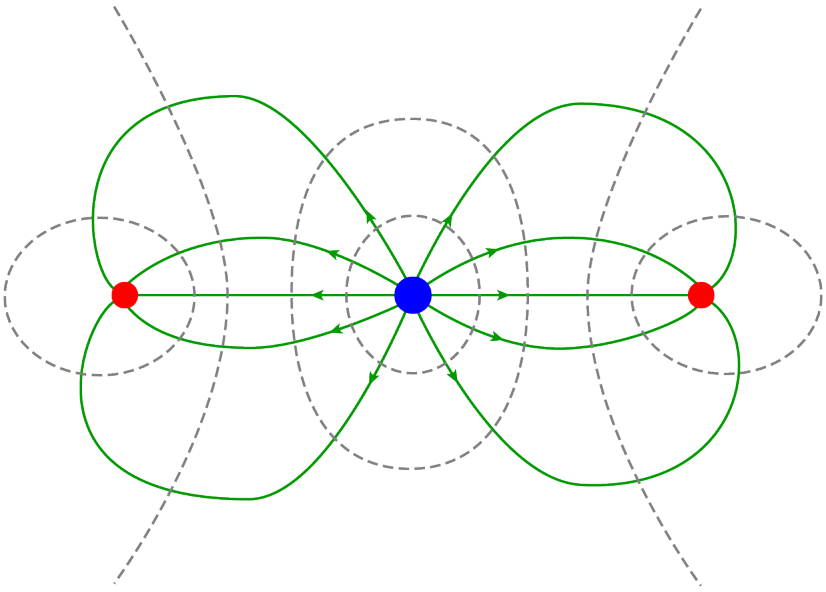
$C = Q/V$, or equivalently $Q = CV$.

B. Energy stored in a capacitor

$U = Q^2/(2C)$. (Using $Q = CV$, this is equivalent to $\frac{1}{2}QV$ and $\frac{1}{2}CV^2$, but the question asked for the form in terms of Q and C.)

2. Electric charges in space

The field lines begin on positive charge and end on negative charge. The central charge $+2Q$ is twice as large in magnitude as each outer charge $-Q$, so twice as many lines should leave it as arrive at each of the outer charges. In the drawing below, ten lines leave the central charge and five arrive at each outer charge. Field lines never cross, and they carry arrows pointing away from the positive charge and toward the negative charges. The field is stronger where the lines are closer together: near the charges, and weaker far from them. Surfaces of constant electric potential are perpendicular to field lines; otherwise the field would do work on a charge moving from one location to another on the surface.



Blue: $+2Q$. Red: $-Q$. Solid green: field lines. Dashed gray: equipotentials.
(This is a sketch, not a calculated plot. The spacing of the lines and the angles where they cross are approximate.)

A. Field lines (solid green)

See the solid lines in the figure. Each line starts on the central charge and ends on one of the outer charges. The line along the axis between charges runs straight, and the other lines curve. Some lines from the center loop around and arrive at the far sides of the outer charges.

B. Surfaces of constant potential (dashed gray)

See the dashed lines in the sketch. Equipotential surfaces are perpendicular to the field lines everywhere. They form closed curves around each charge. If neighboring surfaces differ by the same potential difference, they are closest together where the field is strongest, near the charges.

3. Charged sphere

Outside a spherical charge distribution, the field and potential are the same as for a point charge at the center bearing the whole charge.

A. Potential outside the sphere

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

B. Potential at the surface

Same formula, with $r = R$:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

C. Potential inside the sphere

Inside a conductor in equilibrium the electric field is zero, so the potential does not change with position. It has the same value as at the surface.

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

for all $r < R$.

D. Capacitance of the sphere

The potential is directly proportional to the charge, as it is for any capacitor. Dividing Q by V:

$$C = \frac{Q}{V} = Q\,\frac{4\pi\epsilon_0 R}{Q} = 4\pi\epsilon_0 R$$

The capacitance in farads is directly proportional to the radius in meters, so ϵ_0 has units of farads per meter.

E. Capacitance of a sphere of radius 2.00 m

$C = 4\pi\epsilon_0 R = 4\pi(8.854 \times 10^{-12}\,\mathrm{F/m})(2.00\,\mathrm{m}) = 2.23 \times 10^{-10}\,\mathrm{F} = 223\,\mathrm{pF}$.

That is a small capacitance for such a large device. The separation between the charge on the sphere and the opposite charge (infinitely far away) is very large.

4. Parallel-plate capacitor

The plates have radius $r = 0.250$ m, so the plate area is $A = \pi r^2 = 0.196\,\mathrm{m}^2$. The gap is $d = 1.20 \times 10^{-4}$ m, the dielectric constant is $\kappa = 5$, and the dielectric strength is 12 kV/mm $= 1.2 \times 10^7$ V/m.

A. Capacitance

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{5(8.854 \times 10^{-12}\,\mathrm{F/m})\pi(0.250\,\mathrm{m})^2}{1.20 \times 10^{-4}\,\mathrm{m}} = 7.24 \times 10^{-8}\,\mathrm{F} = 72.4\,\mathrm{nF}$$

B. Charge density on the plates

$$\sigma = \frac{Q}{A} = \frac{14.5 \times 10^{-6}\,\mathrm{C}}{\pi(0.250\,\mathrm{m})^2} = 7.38 \times 10^{-5}\,\mathrm{C/m^2}$$

C. Field outside the dielectric

This is the field that would exist between the plates with no dielectric, produced by the charge on the plates alone:

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{7.38 \times 10^{-5}\,\mathrm{C/m^2}}{8.854 \times 10^{-12}\,\mathrm{C^2/(N{\cdot}m^2)}} = 8.34 \times 10^{6}\,\mathrm{N/C}$$

D. Field inside the dielectric

The dielectric reduces the field by a factor of κ:

$$E = \frac{E_0}{\kappa} = \frac{8.34 \times 10^{6}\,\mathrm{N/C}}{5} = 1.67 \times 10^{6}\,\mathrm{N/C}$$

E. Breakdown voltage

The dielectric strength is the largest field the dielectric can withstand, $E_{\max} = 1.2 \times 10^{7}$ V/m. The potential difference across the gap is the field in the gap times the plate separation, so

$$V_{\max} = E_{\max} d = (1.2 \times 10^{7}\,\mathrm{V/m})(1.20 \times 10^{-4}\,\mathrm{m}) = 1440\,\mathrm{V} = 1.44\,\mathrm{kV}$$

As a check, the present charge of 14.5 μC gives $V = Q/C = 200$ V, well below this limit.