PHYS 1220 Round 2 Retests

Brief Solutions

Standard 7 Retest 1

2. Temperature change of an ideal gas at constant volume

   The question didn't specify that it was at constant volume—at least, not until I edited the question during the retest time—but it was supposed to be. I hope you got word of that clarification before you submitted your answer.

   The gas was diatomic, so its internal energy $U = 5/2\, nRT$. You were told the constant volume V, the initial temperature T_1, and the initial pressure p_1. The gas con't do any pV work as it absorbs heat, so $Q - 0 = \Delta U = 5/2\, nR\Delta T$. We want to find the final temperature.

$$Q = 5/2\, nR(T_2 - T_1)$$
$$T_2 - T_1 = 2/5\, \frac{Q}{nR}$$
$$T_2 = T_1 + \frac{QT_1}{p_1 V}$$

   From there, getting the answer is a matter of plugging in the proper numbers.

   I thought I had included a constant pressure question, but oh well. If you can do one you can probably do the other.

Standard 8 Retest 1

2. Response to heating at 10^5 Pa from 150 K

   The ice would increase in temperature until it reached the melting temperature. At that point, the temperature would remain constant until all the ice melted.

3. Critical point

   At pressures and temperatures beyond the critical point, there is no phase change between liquid and vapor phases. At high temperatures, the liquid density decreases; likewise, at high pressures, the gas density increases. At the critical point, there is no distinction.

4. Triple point pressure

   This question was finding if you could read a logarithmic plot. The triple point plots a little below the 10^3 pascal line. If you estimate it at $10^{2.9}$ Pa, that gives a result around 800 Pa,

5. Triple point temperature

   The horizontal axis of the plot is linear rather than logarithmic, so reading it is straightforward. It is around 275 kelvin.

Standard 9 Retest 1

2. Work is a path function, not a state function , so different paths between the same states can do different amounts of work.

3. Heat is also a path function, so paths between the same states can involve different amounts of heat.

4. Internal energy is a state function, with only one value at every state. Its change depends only on the initial and final states, not the particular path between them

5. State variables are pressure, volume, temperature, and number of moles (of each substance).

Standard 10 Retest 1

Questions 2 and 3 refer to the same process in which a diatomic ideal gas undergoes a constant-temperature expansion with $T = 150°\text{C}$, $p_1 = 600 \times 10^3$ Pa, $V_1 = 10^{-4}$ m^3, and $V_2 = 5 \times 10^{-4}$ m^3.

2. Work done

   Here the work done can be determined by integrating $dW = pdV$ as p and V change.

$$\begin{aligned} dW &= pdV \\ dW &= nRT\frac{dV}{V} \\ W &= nRT\int_{V_1}^{V_2}\frac{dV}{V} \\ &= nRT\ln(V_2/V_1) \\ &= p_1V_1\ln(V_2/V_1) \\ &= 96.57 \text{ J} \end{aligned}$$

3. Heat absorbed

   The internal energy of an ideal gas depends only on temperature. Since this process is isothermal, $\Delta U = 0$, and then the first law of thermodynamics tells us that $Q = W$.

4. Temperature change of a diatomic ideal gas after adiabatic expansion

   We are given the initial temperature T_1 and pressure p_1 as well as the expansion ratio V_2/V_1. We need to find the final temperature. The internal energy of the gas, which uniquely determines the temperature, changes by the amount of work done.

   From the first law of thermodynamics, $U = 5/2\,nRT$, so $dU = 5/2\,nRdT$. Because the process is adiabatic, $dU = 0 - dW = -pdV$. This yields

$$\begin{aligned} f/2\,nRdT &= -\frac{nRT}{V}dV \\ f/2\,\frac{dT}{T} &= -\frac{V}{dV} \\ f/2\int_{T_1}^{T_2}\frac{dT}{T} &= -\int_{V_1}^{V_2}\frac{dV}{V} \\ f/2\,\ln(T_2/T_1) &= \ln(V_1/V_2) \\ (T_2/T_1)^{f/2} &= V_1/V_2 \\ T_2/T_1 &= (V_1/V_2)^{2/f} \\ T_2 &= T_1(V_1/V_2)^{2/f} \end{aligned}$$

5. Internal energy change for an isochoric process

   With zero volume change, no pV work is done, so the internal energy change equals the heat absorbed.

Standard 11 Retest 1

2. Preference for disorder

   The multiplicity of "disorder" is much greater than the multiplicity of "order."

3. Heat transfer down a temperature gradient

   An energy change makes a bigger entropy difference at low temperature. So transferring heat from high temperature to low temperature is an entropic winner.

Standard 12 Retest 1

Both questions in this quiz consisted of calculating entropy changes bnetween the same initial and final states. The system was a diatomic ideal gas with initial temperature $150°\text{C} = 423.15$ K, initial pressure 600×10^3 Pa, and initial volume 1.000×10^{-4} m^3. The final state had the same temperature but five times the volume.

2. Isothermal expansion from 100 mL to 500 mL

   The first path was an isothermal expansion. Since the temperature is constant, we can use the formula $\Delta S = Q/T$ to calculate the entropy change. The heat absorbed in the isothermal expansion of an ideal gas is $Q = p_1 V_1 \ln(V_2/V_1)$, so

   $$\Delta S = \frac{p_1 V_1}{T} \ln(V_2/V_1) = 0.228 \text{ J/K}.$$

3. Free expansion from 100 mL to 500 mL

   Here we just use the formula

   $$\Delta S = nR \ln(V_2/V_1) = \frac{p_1 V_1}{T} \ln(V_2/V_1) = 0.228 \text{ J/K}.$$

   Surprisingly, or perhaps not, this is exactly the same formula, and consequently the same result, as was obtained from the isothermal expansion. This underscores the fact that entropy is a state function.