PHYS 1220 Group Work Sheet
Heat transfer

Key Ideas

The heat current through slab of material with conductivity k_t, area A, thickness Δx, and temperature drop ΔT is given by the **Fourier heat conduction law** $\frac{dQ}{dt} = -k_t A \frac{\Delta T}{\Delta x}$.

Conductivities k_t for several materials in W/(m·K) include air 0.026, wood 0.08, water 0.6, glass 0.8, ice 2.30, iron 80, and copper 400.

The energy emitted as electromagnetic radiation from a hot surface of area A is given by the **Stefan-Boltzmann law** $dQ/dt = Ae\sigma T^4$. The **emissivity** e of the emitter is a number between 0 and 1, matching its absorbtivity. The fundamental Stefan-Boltmann constant $\sigma = 5.67037442\times10^{-8}$ W/(m^2·K^4).

With your group, discuss how to answer these questions and write your group answer in the space provided. Explain/show how you got your answer: don't just write down a number! (Especially not one without units!)

1. A lake is covered by a sheet of ice 1.0 cm thick. The air above the ice is –10 °C and the water below the ice is 0 °C.
   a. Does it make sense to quantify the rate of heat transfer through the ice as dQ/dt? Is there a better quantity to characterize the process?

   b. What is the rate of heat transfer through the ice? Use the metric you chose in part a.

   c. At what rate is the ice getting thicker? The latent heat of melting ice is 335 kJ/kg. The density of ice is 920 kg/m^3.

d. As the ice thickens, this rate changes; you can figure out the thickness and rate of freezing at any time by setting up and solving a differential equation. What is that equation?

e. Can you solve the differential equation?

2. The radius of the Sun is 6.96×10^8 meters and its average surface temperature is 5772 K.

   a. What is the Sun's surface area, assuming it is a perfect sphere?

   b. Assuming that the Sun is a perfect black body, how much energy does it radiate per square meter of surface?

   c. What is the total power output of the Sun?

   d. By the relativistic formula $E = mc^2$, at what rate is matter converted to energy within the Sun?

   e. The Sun's mass is 1.99×10^{30} kg. What is its proportional rate of mass conversion $\frac{1}{m}\frac{dm}{dt}$?